

8 Similarity

- 8.1 Dilations
- 8.2 Similarity and Transformations
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Magnification (p. 467)



Photo Stickers (p. 465)

Maintaining Mathematical Proficiency

Determining Whether Ratios Form a Proportion

Example 1 Tell whether $\frac{2}{8}$ and $\frac{3}{12}$ form a proportion.

Compare the ratios in simplest form.

$$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4} \qquad \frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

The ratios are equivalent.

▶ So, $\frac{2}{8}$ and $\frac{3}{12}$ form a proportion.

Tell whether the ratios form a proportion.

1. $\frac{5}{3}, \frac{35}{21}$

2. $\frac{9}{24}, \frac{24}{64}$

3. $\frac{8}{56}, \frac{6}{28}$

4. $\frac{18}{4}, \frac{27}{9}$

5. $\frac{15}{21}, \frac{55}{77}$

6. $\frac{26}{8}, \frac{39}{12}$

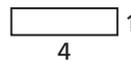
Identifying Similar Figures

Example 2 Which rectangle is similar to Rectangle A?

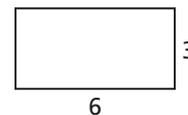
Rectangle A



Rectangle B



Rectangle C



Each figure is a rectangle, so corresponding angles are congruent. Check to see whether corresponding side lengths are proportional.

Rectangle A and Rectangle B

$$\frac{\text{Length of A}}{\text{Length of B}} = \frac{8}{4} = 2 \qquad \frac{\text{Width of A}}{\text{Width of B}} = \frac{4}{1} = 4$$

not proportional

Rectangle A and Rectangle C

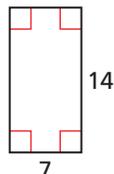
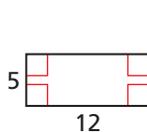
$$\frac{\text{Length of A}}{\text{Length of C}} = \frac{8}{6} = \frac{4}{3} \qquad \frac{\text{Width of A}}{\text{Width of C}} = \frac{4}{3}$$

proportional

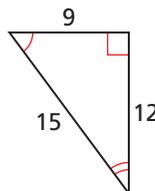
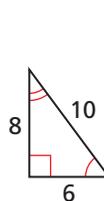
▶ So, Rectangle C is similar to Rectangle A.

Tell whether the two figures are similar. Explain your reasoning.

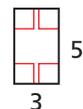
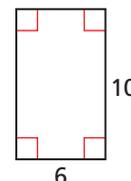
7.



8.



9.



10. **ABSTRACT REASONING** Can you draw two squares that are not similar? Explain your reasoning.

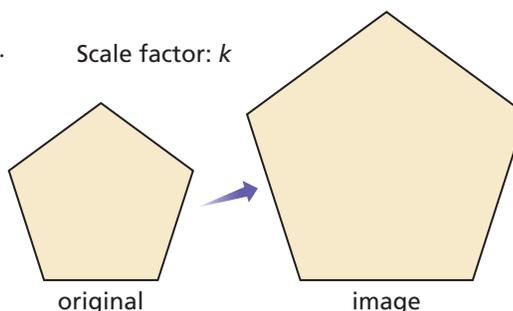
Discerning a Pattern or Structure

Core Concept

Dilations, Perimeter, Area, and Volume

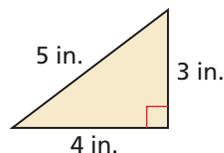
Consider a figure that is dilated by a scale factor of k .

1. The perimeter of the image is k times the perimeter of the original figure.
2. The area of the image is k^2 times the area of the original figure.
3. If the original figure is three dimensional, then the volume of the image is k^3 times the volume of the original figure.



EXAMPLE 1 Finding Perimeter and Area after a Dilation

The triangle shown has side lengths of 3 inches, 4 inches, and 5 inches. Find the perimeter and area of the image when the triangle is dilated by a scale factor of (a) 2, (b) 3, and (c) 4.



SOLUTION

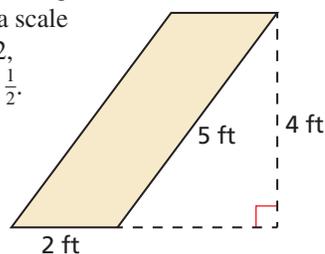
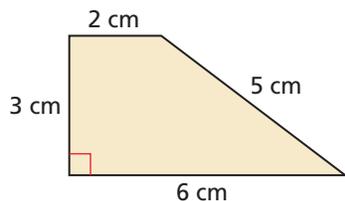
Perimeter: $P = 5 + 3 + 4 = 12$ in.

Area: $A = \frac{1}{2}(4)(3) = 6$ in.²

	Scale factor: k	Perimeter: kP	Area: k^2A
a.	2	$2(12) = 24$ in.	$(2^2)(6) = 24$ in. ²
b.	3	$3(12) = 36$ in.	$(3^2)(6) = 54$ in. ²
c.	4	$4(12) = 48$ in.	$(4^2)(6) = 96$ in. ²

Monitoring Progress

1. Find the perimeter and area of the image when the trapezoid is dilated by a scale factor of (a) 2, (b) 3, and (c) 4.
2. Find the perimeter and area of the image when the parallelogram is dilated by a scale factor of (a) $\frac{1}{2}$, (b) 3, and (c) $\frac{1}{2}$.



3. A rectangular prism is 3 inches wide, 4 inches long, and 5 inches tall. Find the surface area and volume of the image of the prism when it is dilated by a scale factor of (a) 2, (b) 3, and (c) 4.

8.1 Dilations

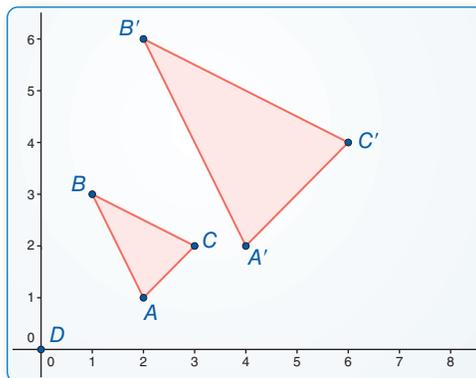
Essential Question

What does it mean to dilate a figure?

EXPLORATION 1 Dilating a Triangle in a Coordinate Plane

Work with a partner. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

- a. Dilate $\triangle ABC$ using a *scale factor* of 2 and a *center of dilation* at the origin to form $\triangle A'B'C'$. Compare the coordinates, side lengths, and angle measures of $\triangle ABC$ and $\triangle A'B'C'$.



Sample

Points

$$A(2, 1)$$

$$B(1, 3)$$

$$C(3, 2)$$

Segments

$$AB = 2.24$$

$$BC = 2.24$$

$$AC = 1.41$$

Angles

$$m\angle A = 71.57^\circ$$

$$m\angle B = 36.87^\circ$$

$$m\angle C = 71.57^\circ$$

LOOKING FOR STRUCTURE

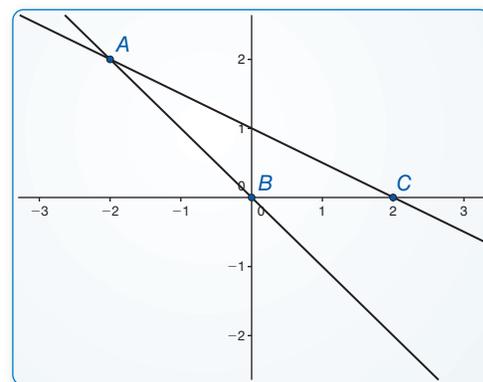
To be proficient in math, you need to look closely to discern a pattern or structure.

- b. Repeat part (a) using a *scale factor* of $\frac{1}{2}$.
- c. What do the results of parts (a) and (b) suggest about the coordinates, side lengths, and angle measures of the image of $\triangle ABC$ after a dilation with a scale factor of k ?

EXPLORATION 2 Dilating Lines in a Coordinate Plane

Work with a partner. Use dynamic geometry software to draw \overleftrightarrow{AB} that passes through the origin and \overleftrightarrow{AC} that does not pass through the origin.

- a. Dilate \overleftrightarrow{AB} using a *scale factor* of 3 and a *center of dilation* at the origin. Describe the image.
- b. Dilate \overleftrightarrow{AC} using a *scale factor* of 3 and a *center of dilation* at the origin. Describe the image.
- c. Repeat parts (a) and (b) using a scale factor of $\frac{1}{4}$.
- d. What do you notice about dilations of lines passing through the center of dilation and dilations of lines not passing through the center of dilation?



Sample

Points

$$A(-2, 2)$$

$$B(0, 0)$$

$$C(2, 0)$$

Lines

$$x + y = 0$$

$$x + 2y = 2$$

Communicate Your Answer

3. What does it mean to dilate a figure?
4. Repeat Exploration 1 using a center of dilation at a point other than the origin.

8.1 Lesson

Core Vocabulary

dilation, p. 462
 center of dilation, p. 462
 scale factor, p. 462
 enlargement, p. 462
 reduction, p. 462

What You Will Learn

- ▶ Identify and perform dilations.
- ▶ Solve real-life problems involving scale factors and dilations.

Identifying and Performing Dilations

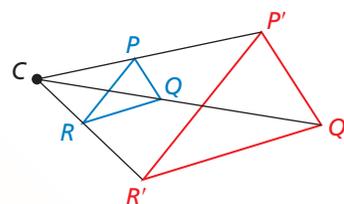
Core Concept

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k , which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center point C , then $P = P'$.
- If P is not the center point C , then the image point P' lies on \overline{CP} . The scale factor k is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.

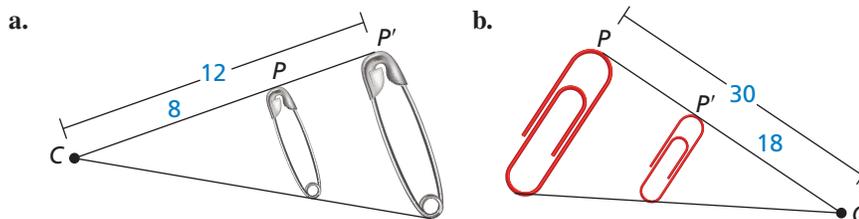


A dilation does not change any line that passes through the center of dilation. A dilation maps a line that does not pass through the center of dilation to a parallel line. In the figure above, $\overrightarrow{PR} \parallel \overrightarrow{P'R'}$, $\overrightarrow{PQ} \parallel \overrightarrow{P'Q'}$, and $\overrightarrow{QR} \parallel \overrightarrow{Q'R'}$.

When the scale factor $k > 1$, a dilation is an **enlargement**. When $0 < k < 1$, a dilation is a **reduction**.

EXAMPLE 1 Identifying Dilations

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.



SOLUTION

- a. Because $\frac{CP'}{CP} = \frac{12}{8}$, the scale factor is $k = \frac{3}{2}$. So, the dilation is an enlargement.
- b. Because $\frac{CP'}{CP} = \frac{18}{30}$, the scale factor is $k = \frac{3}{5}$. So, the dilation is a reduction.

READING

The scale factor of a dilation can be written as a fraction, decimal, or percent.

Monitoring Progress



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1. In a dilation, $CP' = 3$ and $CP = 12$. Find the scale factor. Then tell whether the dilation is a *reduction* or an *enlargement*.

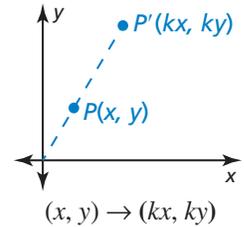
Core Concept

READING DIAGRAMS

In this chapter, for all of the dilations in the coordinate plane, the center of dilation is the origin unless otherwise noted.

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.



EXAMPLE 2 Dilating a Figure in the Coordinate Plane

Graph $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 1)$, and $C(4, -1)$ and its image after a dilation with a scale factor of 2.

SOLUTION

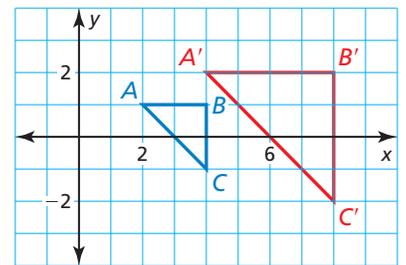
Use the coordinate rule for a dilation with $k = 2$ to find the coordinates of the vertices of the image. Then graph $\triangle ABC$ and its image.

$$(x, y) \rightarrow (2x, 2y)$$

$$A(2, 1) \rightarrow A'(4, 2)$$

$$B(4, 1) \rightarrow B'(8, 2)$$

$$C(4, -1) \rightarrow C'(8, -2)$$



Notice the relationships between the lengths and slopes of the sides of the triangles in Example 2. Each side length of $\triangle A'B'C'$ is longer than its corresponding side by the scale factor. The corresponding sides are parallel because their slopes are the same.

EXAMPLE 3 Dilating a Figure in the Coordinate Plane

Graph quadrilateral $KLMN$ with vertices $K(-3, 6)$, $L(0, 6)$, $M(3, 3)$, and $N(-3, -3)$ and its image after a dilation with a scale factor of $\frac{1}{3}$.

SOLUTION

Use the coordinate rule for a dilation with $k = \frac{1}{3}$ to find the coordinates of the vertices of the image. Then graph quadrilateral $KLMN$ and its image.

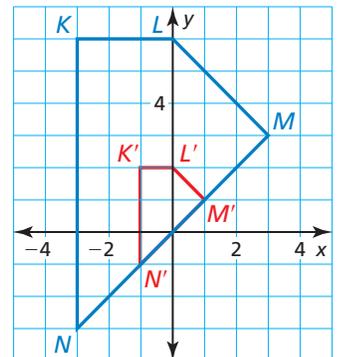
$$(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$$

$$K(-3, 6) \rightarrow K'(-1, 2)$$

$$L(0, 6) \rightarrow L'(0, 2)$$

$$M(3, 3) \rightarrow M'(1, 1)$$

$$N(-3, -3) \rightarrow N'(-1, -1)$$



Monitoring Progress



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Graph $\triangle PQR$ and its image after a dilation with scale factor k .

2. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$

3. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

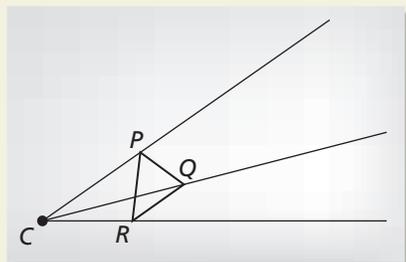
CONSTRUCTION

Constructing a Dilation

Use a compass and straightedge to construct a dilation of $\triangle PQR$ with a scale factor of 2. Use a point C outside the triangle as the center of dilation.

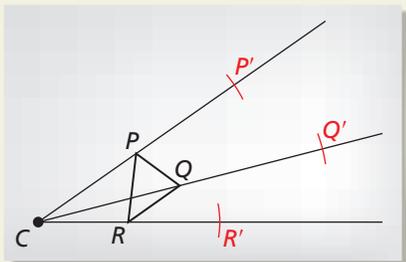
SOLUTION

Step 1



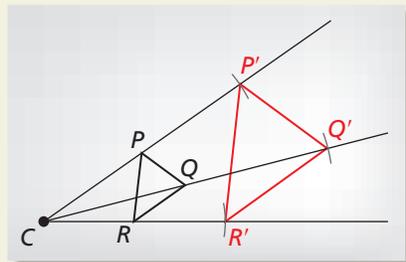
Draw a triangle Draw $\triangle PQR$ and choose the center of the dilation C outside the triangle. Draw rays from C through the vertices of the triangle.

Step 2

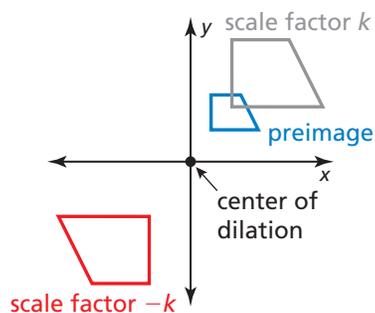


Use a compass Use a compass to locate P' on CP so that $CP' = 2(CP)$. Locate Q' and R' using the same method.

Step 3



Connect points Connect points P' , Q' , and R' to form $\triangle P'Q'R'$.



In the coordinate plane, you can have scale factors that are negative numbers. When this occurs, the figure rotates 180° . So, when $k > 0$, a dilation with a scale factor of $-k$ is the same as the composition of a dilation with a scale factor of k followed by a rotation of 180° about the center of dilation. Using the coordinate rules for a dilation and a rotation of 180° , you can think of the notation as

$$(x, y) \rightarrow (kx, ky) \rightarrow (-kx, -ky).$$

EXAMPLE 4

Using a Negative Scale Factor

Graph $\triangle FGH$ with vertices $F(-4, -2)$, $G(-2, 4)$, and $H(-2, -2)$ and its image after a dilation with a scale factor of $-\frac{1}{2}$.

SOLUTION

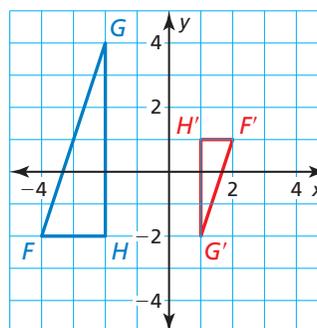
Use the coordinate rule for a dilation with $k = -\frac{1}{2}$ to find the coordinates of the vertices of the image. Then graph $\triangle FGH$ and its image.

$$(x, y) \rightarrow \left(-\frac{1}{2}x, -\frac{1}{2}y\right)$$

$$F(-4, -2) \rightarrow F'(2, 1)$$

$$G(-2, 4) \rightarrow G'(1, -2)$$

$$H(-2, -2) \rightarrow H'(1, 1)$$



Monitoring Progress



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- Graph $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 1)$, and $R(1, -3)$ and its image after a dilation with a scale factor of -2 .
- Suppose a figure containing the origin is dilated. Explain why the corresponding point in the image of the figure is also the origin.

Solving Real-Life Problems

EXAMPLE 5 Finding a Scale Factor

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?



READING

Scale factors are written so that the units in the numerator and denominator divide out.

SOLUTION

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$.

► So, in simplest form, the scale factor is $\frac{11}{40}$.

EXAMPLE 6 Finding the Length of an Image

You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.



SOLUTION

$$\frac{\text{image length}}{\text{actual length}} = k$$

$$\frac{x}{1.5} = 6$$

$$x = 9$$

► So, the image length through the magnifying glass is 9 centimeters.

Monitoring Progress



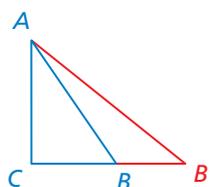
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- An optometrist dilates the pupils of a patient's eyes to get a better look at the back of the eyes. A pupil dilates from 4.5 millimeters to 8 millimeters. What is the scale factor of this dilation?
- The image of a spider seen through the magnifying glass in Example 6 is shown at the left. Find the actual length of the spider.

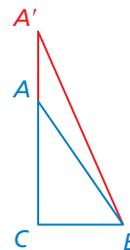


When a transformation, such as a dilation, changes the shape or size of a figure, the transformation is *nonrigid*. In addition to dilations, there are many possible nonrigid transformations. Two examples are shown below. It is important to pay close attention to whether a nonrigid transformation preserves lengths and angle measures.

Horizontal Stretch



Vertical Stretch



8.1 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point _____.
- WHICH ONE DOESN'T BELONG?** Which scale factor does *not* belong with the other three? Explain your reasoning.

$$\frac{5}{4}$$

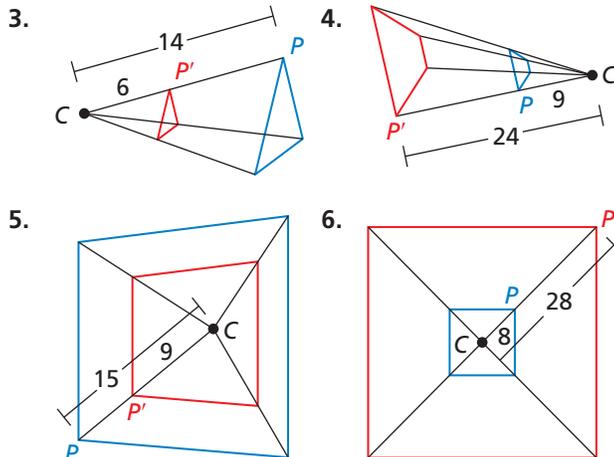
60%

115%

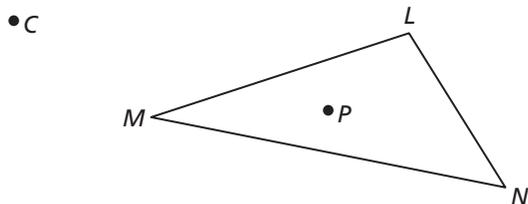
2

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*. (See Example 1.)

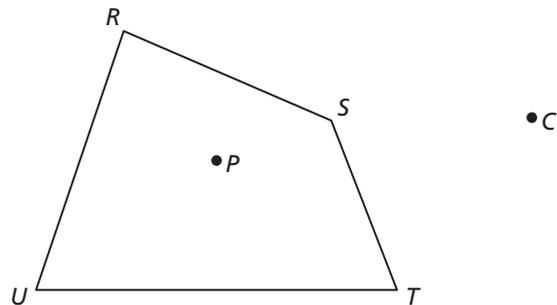


CONSTRUCTION In Exercises 7–10, copy the diagram. Then use a compass and straightedge to construct a dilation of $\triangle LMN$ with the given center and scale factor k .



- Center C , $k = 2$
- Center P , $k = 3$
- Center M , $k = \frac{1}{2}$
- Center C , $k = 25\%$

CONSTRUCTION In Exercises 11–14, copy the diagram. Then use a compass and straightedge to construct a dilation of quadrilateral $RSTU$ with the given center and scale factor k .



- Center C , $k = 3$
- Center P , $k = 2$
- Center R , $k = 0.25$
- Center C , $k = 75\%$

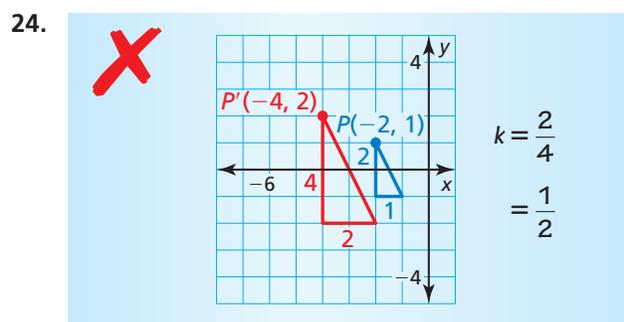
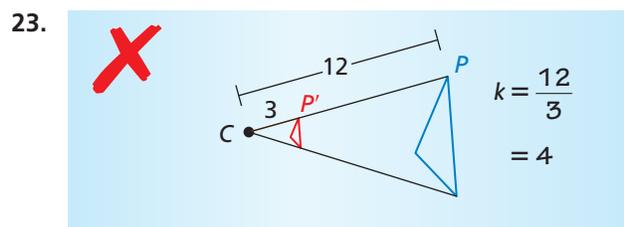
In Exercises 15–18, graph the polygon and its image after a dilation with scale factor k . (See Examples 2 and 3.)

- $X(6, -1)$, $Y(-2, -4)$, $Z(1, 2)$; $k = 3$
- $A(0, 5)$, $B(-10, -5)$, $C(5, -5)$; $k = 120\%$
- $T(9, -3)$, $U(6, 0)$, $V(3, 9)$, $W(0, 0)$; $k = \frac{2}{3}$
- $J(4, 0)$, $K(-8, 4)$, $L(0, -4)$, $M(12, -8)$; $k = 0.25$

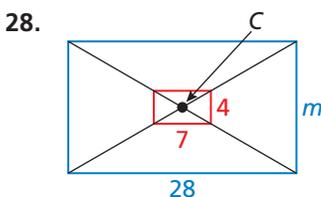
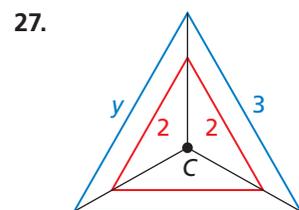
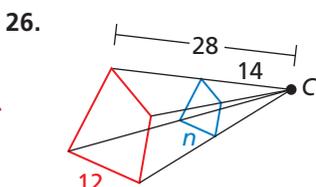
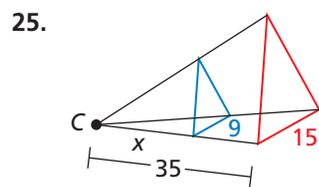
In Exercises 19–22, graph the polygon and its image after a dilation with scale factor k . (See Example 4.)

- $B(-5, -10)$, $C(-10, 15)$, $D(0, 5)$; $k = -\frac{1}{5}$
- $L(0, 0)$, $M(-4, 1)$, $N(-3, -6)$; $k = -3$
- $R(-7, -1)$, $S(2, 5)$, $T(-2, -3)$, $U(-3, -3)$; $k = -4$
- $W(8, -2)$, $X(6, 0)$, $Y(-6, 4)$, $Z(-2, 2)$; $k = -0.5$

ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in finding the scale factor of the dilation.



In Exercises 25–28, the red figure is the image of the blue figure after a dilation with center C . Find the scale factor of the dilation. Then find the value of the variable.

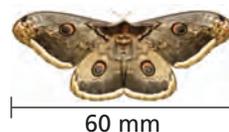


29. **FINDING A SCALE FACTOR** You receive wallet-sized photos of your school picture. The photo is 2.5 inches by 3.5 inches. You decide to dilate the photo to 5 inches by 7 inches at the store. What is the scale factor of this dilation? (See Example 5.)

30. **FINDING A SCALE FACTOR** Your visually impaired friend asked you to enlarge your notes from class so he can study. You took notes on 8.5-inch by 11-inch paper. The enlarged copy has a smaller side with a length of 10 inches. What is the scale factor of this dilation?

In Exercises 31–34, you are using a magnifying glass. Use the length of the insect and the magnification level to determine the length of the image seen through the magnifying glass. (See Example 6.)

31. emperor moth
Magnification: $5\times$



32. ladybug
Magnification: $10\times$



33. dragonfly
Magnification: $20\times$



34. carpenter ant
Magnification: $15\times$



35. **ANALYZING RELATIONSHIPS** Use the given actual and magnified lengths to determine which of the following insects were looked at using the same magnifying glass. Explain your reasoning.

grasshopper
Actual: 2 in.
Magnified: 15 in.



black beetle
Actual: 0.6 in.
Magnified: 4.2 in.



honeybee
Actual: $\frac{5}{8}$ in.
Magnified: $\frac{75}{16}$ in.



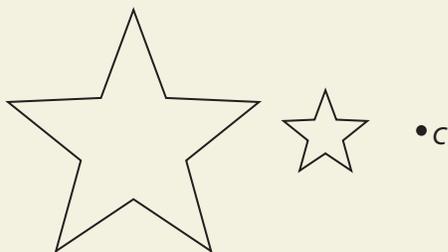
monarch butterfly
Actual: 3.9 in.
Magnified: 29.25 in.



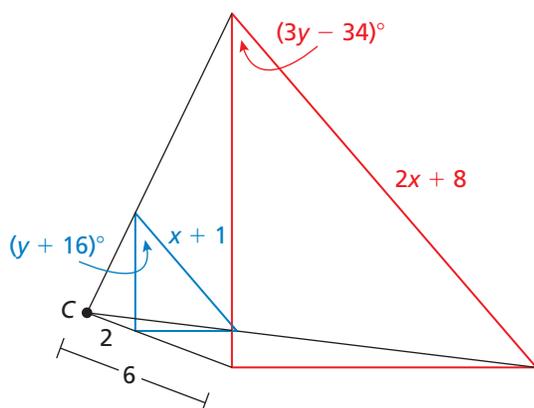
36. **THOUGHT PROVOKING** Draw $\triangle ABC$ and $\triangle A'B'C'$ so that $\triangle A'B'C'$ is a dilation of $\triangle ABC$. Find the center of dilation and explain how you found it.

37. **REASONING** Your friend prints a 4-inch by 6-inch photo for you from the school dance. All you have is an 8-inch by 10-inch frame. Can you dilate the photo to fit the frame? Explain your reasoning.

38. **HOW DO YOU SEE IT?** Point C is the center of dilation of the images. The scale factor is $\frac{1}{3}$. Which figure is the original figure? Which figure is the dilated figure? Explain your reasoning.



39. **MATHEMATICAL CONNECTIONS** The larger triangle is a dilation of the smaller triangle. Find the values of x and y .



40. **WRITING** Explain why a scale factor of 2 is the same as 200%.

In Exercises 41–44, determine whether the dilated figure or the original figure is closer to the center of dilation. Use the given location of the center of dilation and scale factor k .

41. Center of dilation: inside the figure; $k = 3$
 42. Center of dilation: inside the figure; $k = \frac{1}{2}$
 43. Center of dilation: outside the figure; $k = 120\%$
 44. Center of dilation: outside the figure; $k = 0.1$

45. **ANALYZING RELATIONSHIPS** Dilate the line through $O(0, 0)$ and $A(1, 2)$ using a scale factor of 2.

- a. What do you notice about the lengths of $\overline{O'A'}$ and \overline{OA} ?
 b. What do you notice about $\overleftrightarrow{O'A'}$ and \overleftrightarrow{OA} ?

46. **ANALYZING RELATIONSHIPS** Dilate the line through $A(0, 1)$ and $B(1, 2)$ using a scale factor of $\frac{1}{2}$.

- a. What do you notice about the lengths of $\overline{A'B'}$ and \overline{AB} ?
 b. What do you notice about $\overleftrightarrow{A'B'}$ and \overleftrightarrow{AB} ?

47. **ATTENDING TO PRECISION** You are making a blueprint of your house. You measure the lengths of the walls of your room to be 11 feet by 12 feet. When you draw your room on the blueprint, the lengths of the walls are 8.25 inches by 9 inches. What scale factor dilates your room to the blueprint?

48. **MAKING AN ARGUMENT** Your friend claims that dilating a figure by 1 is the same as dilating a figure by -1 because the original figure will not be enlarged or reduced. Is your friend correct? Explain your reasoning.

49. **USING STRUCTURE** Rectangle $WXYZ$ has vertices $W(-3, -1)$, $X(-3, 3)$, $Y(5, 3)$, and $Z(5, -1)$.

- a. Find the perimeter and area of the rectangle.
 b. Dilate the rectangle using a scale factor of 3. Find the perimeter and area of the dilated rectangle. Compare with the original rectangle. What do you notice?
 c. Repeat part (b) using a scale factor of $\frac{1}{4}$.
 d. Make a conjecture for how the perimeter and area change when a figure is dilated.

50. **REASONING** You put a reduction of a page on the original page. Explain why there is a point that is in the same place on both pages.

51. **REASONING** $\triangle ABC$ has vertices $A(4, 2)$, $B(4, 6)$, and $C(7, 2)$. Find the coordinates of the vertices of the image after a dilation with center $(4, 0)$ and a scale factor of 2.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

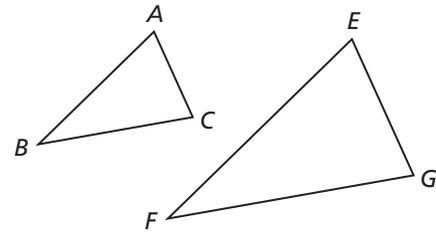
The vertices of $\triangle ABC$ are $A(2, -1)$, $B(0, 4)$, and $C(-3, 5)$. Find the coordinates of the vertices of the image after the translation. (Skills Review Handbook)

52. $(x, y) \rightarrow (x, y - 4)$ 53. $(x, y) \rightarrow (x - 1, y + 3)$ 54. $(x, y) \rightarrow (x + 3, y - 1)$
 55. $(x, y) \rightarrow (x - 2, y)$ 56. $(x, y) \rightarrow (x + 1, y - 2)$ 57. $(x, y) \rightarrow (x - 3, y + 1)$

8.2 Similarity and Transformations

Essential Question When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure?

Two figures are *similar figures* when they have the same shape but not necessarily the same size.



Similar Triangles

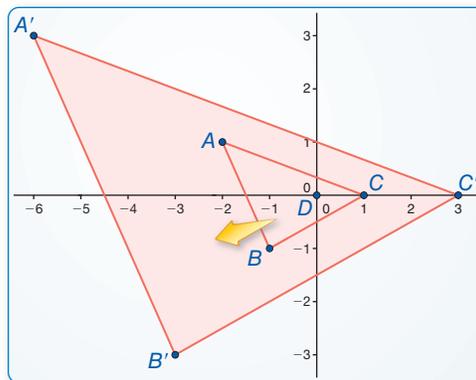
ATTENDING TO PRECISION

To be proficient in math, you need to use clear definitions in discussions with others and in your own reasoning.

EXPLORATION 1 Dilations and Similarity

Work with a partner.

- Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.
- Dilate the triangle using a scale factor of 3. Is the image similar to the original triangle? Justify your answer.



Sample

Points

$A(-2, 1)$

$B(-1, -1)$

$C(1, 0)$

$D(0, 0)$

Segments

$AB = 2.24$

$BC = 2.24$

$AC = 3.16$

Angles

$m\angle A = 45^\circ$

$m\angle B = 90^\circ$

$m\angle C = 45^\circ$

EXPLORATION 2 Rigid Motions and Similarity

Work with a partner.

- Use dynamic geometry software to draw any triangle.
- Copy the triangle and translate it 3 units left and 4 units up. Is the image similar to the original triangle? Justify your answer.
- Reflect the triangle in the y -axis. Is the image similar to the original triangle? Justify your answer.
- Rotate the original triangle 90° counterclockwise about the origin. Is the image similar to the original triangle? Justify your answer.

Communicate Your Answer

- When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure? Explain your reasoning.
- A figure undergoes a composition of transformations, which includes translations, reflections, rotations, and dilations. Is the image similar to the original figure? Explain your reasoning.

8.2 Lesson

Core Vocabulary

similarity transformation,
p. 470
similar figures, p. 470

What You Will Learn

- ▶ Perform similarity transformations.
- ▶ Describe similarity transformations.
- ▶ Prove that figures are similar.

Performing Similarity Transformations

A dilation is a transformation that preserves shape but not size. So, a dilation is a nonrigid motion. A **similarity transformation** is a dilation or a composition of rigid motions and dilations. Two geometric figures are **similar figures** if and only if there is a similarity transformation that maps one of the figures onto the other. Similar figures have the same shape but not necessarily the same size.

Congruence transformations preserve length and angle measure. When the scale factor of the dilation(s) is not equal to 1 or -1 , similarity transformations preserve angle measure only.

EXAMPLE 1 Performing a Similarity Transformation

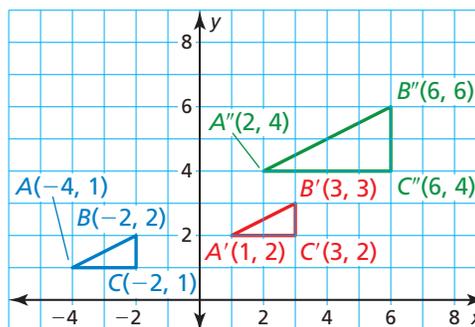
Graph $\triangle ABC$ with vertices $A(-4, 1)$, $B(-2, 2)$, and $C(-2, 1)$ and its image after the similarity transformation.

Translation: $(x, y) \rightarrow (x + 5, y + 1)$

Dilation: $(x, y) \rightarrow (2x, 2y)$

SOLUTION

Step 1 Graph $\triangle ABC$.



Step 2 Translate $\triangle ABC$ 5 units right and 1 unit up. $\triangle A'B'C'$ has vertices $A'(1, 2)$, $B'(3, 3)$, and $C'(3, 2)$.

Step 3 Dilate $\triangle A'B'C'$ using a scale factor of 2. $\triangle A''B''C''$ has vertices $A''(2, 4)$, $B''(6, 6)$, and $C''(6, 4)$.

Monitoring Progress



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1. Graph \overline{CD} with endpoints $C(-2, 2)$ and $D(2, 2)$ and its image after the similarity transformation.

Rotation: 90° about the origin

Dilation: $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

2. Graph $\triangle FGH$ with vertices $F(1, 2)$, $G(4, 4)$, and $H(2, 0)$ and its image after the similarity transformation.

Reflection: in the x -axis

Dilation: $(x, y) \rightarrow (1.5x, 1.5y)$

STUDY TIP

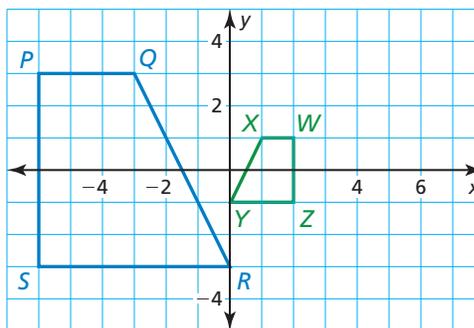
In this chapter, all rotations are counterclockwise unless otherwise noted.

Describing Similarity Transformations

EXAMPLE 2

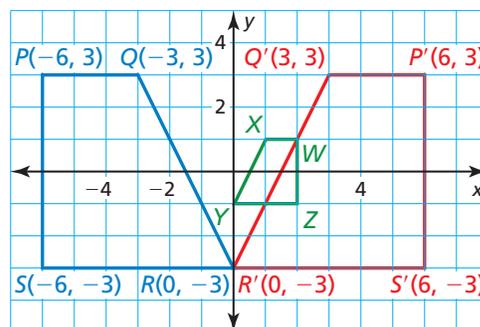
 Describing a Similarity Transformation

Describe a similarity transformation that maps trapezoid $PQRS$ to trapezoid $WXYZ$.



SOLUTION

\overline{QR} falls from left to right, and \overline{XY} rises from left to right. If you reflect trapezoid $PQRS$ in the y -axis as shown, then the image, trapezoid $P'Q'R'S'$, will have the same orientation as trapezoid $WXYZ$.



Trapezoid $WXYZ$ appears to be about one-third as large as trapezoid $P'Q'R'S'$. Dilate trapezoid $P'Q'R'S'$ using a scale factor of $\frac{1}{3}$.

$$(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$$

$$P'(6, 3) \rightarrow P''(2, 1)$$

$$Q'(3, 3) \rightarrow Q''(1, 1)$$

$$R'(0, -3) \rightarrow R''(0, -1)$$

$$S'(6, -3) \rightarrow S''(2, -1)$$

The vertices of trapezoid $P''Q''R''S''$ match the vertices of trapezoid $WXYZ$.

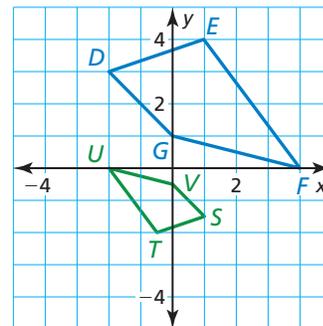
► So, a similarity transformation that maps trapezoid $PQRS$ to trapezoid $WXYZ$ is a reflection in the y -axis followed by a dilation with a scale factor of $\frac{1}{3}$.

Monitoring Progress



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3. In Example 2, describe another similarity transformation that maps trapezoid $PQRS$ to trapezoid $WXYZ$.
4. Describe a similarity transformation that maps quadrilateral $DEFG$ to quadrilateral $STUV$.



Proving Figures Are Similar

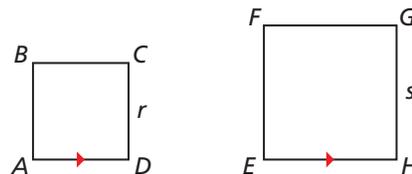
To prove that two figures are similar, you must prove that a similarity transformation maps one of the figures onto the other.

EXAMPLE 3 Proving That Two Squares Are Similar

Prove that square $ABCD$ is similar to square $EFGH$.

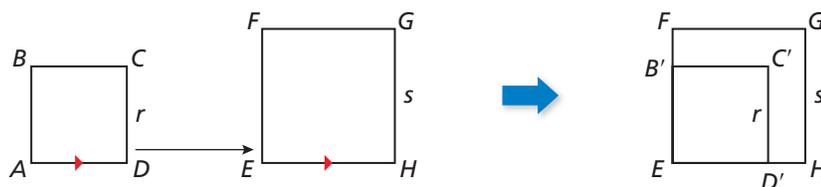
Given Square $ABCD$ with side length r ,
square $EFGH$ with side length s ,
 $\overline{AD} \parallel \overline{EH}$

Prove Square $ABCD$ is similar to
square $EFGH$.

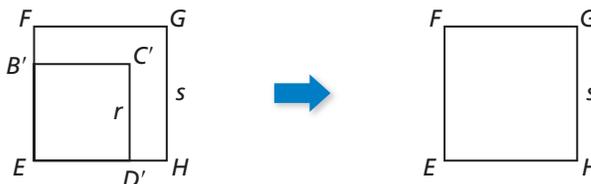


SOLUTION

Translate square $ABCD$ so that point A maps to point E . Because translations map segments to parallel segments and $\overline{AD} \parallel \overline{EH}$, the image of \overline{AD} lies on \overline{EH} .

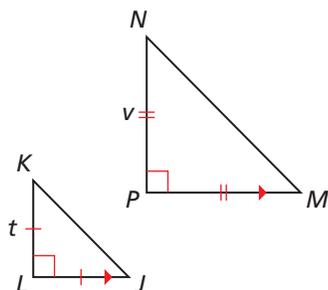


Because translations preserve length and angle measure, the image of $ABCD$, $EB'C'D'$, is a square with side length r . Because all the interior angles of a square are right angles, $\angle B'ED' \cong \angle FEH$. When $\overrightarrow{ED'}$ coincides with \overrightarrow{EH} , $\overrightarrow{EB'}$ coincides with \overrightarrow{EF} . So, $\overline{EB'}$ lies on \overline{EF} . Next, dilate square $EB'C'D'$ using center of dilation E . Choose the scale factor to be the ratio of the side lengths of $EFGH$ and $EB'C'D'$, which is $\frac{s}{r}$.



This dilation maps $\overline{ED'}$ to \overline{EH} and $\overline{EB'}$ to \overline{EF} because the images of $\overline{ED'}$ and $\overline{EB'}$ have side length $\frac{s}{r}(r) = s$ and the segments $\overline{ED'}$ and $\overline{EB'}$ lie on lines passing through the center of dilation. So, the dilation maps B' to F and D' to H . The image of C' lies $\frac{s}{r}(r) = s$ units to the right of the image of B' and $\frac{s}{r}(r) = s$ units above the image of D' . So, the image of C' is G .

▶ A similarity transformation maps square $ABCD$ to square $EFGH$. So, square $ABCD$ is similar to square $EFGH$.



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5. Prove that $\triangle JKL$ is similar to $\triangle MNP$.

Given Right isosceles $\triangle JKL$ with leg length t , right isosceles $\triangle MNP$ with leg length v , $\overline{LJ} \parallel \overline{PM}$

Prove $\triangle JKL$ is similar to $\triangle MNP$.

8.2 Exercises

Vocabulary and Core Concept Check

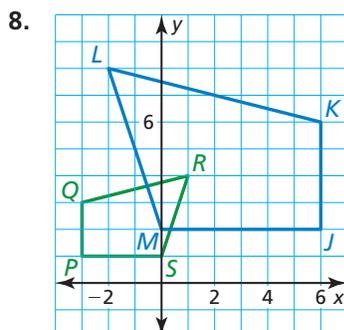
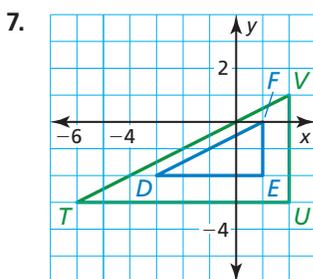
- VOCABULARY** What is the difference between *similar figures* and *congruent figures*?
- COMPLETE THE SENTENCE** A transformation that produces a similar figure, such as a dilation, is called a _____.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph $\triangle FGH$ with vertices $F(-2, 2)$, $G(-2, -4)$, and $H(-4, -4)$ and its image after the similarity transformation. (See Example 1.)

- Translation:** $(x, y) \rightarrow (x + 3, y + 1)$
Dilation: $(x, y) \rightarrow (2x, 2y)$
- Dilation:** $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$
Reflection: in the y -axis
- Rotation:** 90° about the origin
Dilation: $(x, y) \rightarrow (3x, 3y)$
- Dilation:** $(x, y) \rightarrow (\frac{3}{4}x, \frac{3}{4}y)$
Reflection: in the x -axis

In Exercises 7 and 8, describe a similarity transformation that maps the blue preimage to the green image. (See Example 2.)



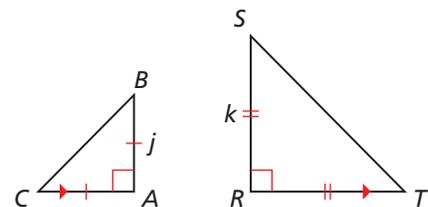
In Exercises 9–12, determine whether the polygons with the given vertices are similar. Use transformations to explain your reasoning.

- $A(6, 0)$, $B(9, 6)$, $C(12, 6)$ and $D(0, 3)$, $E(1, 5)$, $F(2, 5)$
- $Q(-1, 0)$, $R(-2, 2)$, $S(1, 3)$, $T(2, 1)$ and $W(0, 2)$, $X(4, 4)$, $Y(6, -2)$, $Z(2, -4)$
- $G(-2, 3)$, $H(4, 3)$, $I(4, 0)$ and $J(1, 0)$, $K(6, -2)$, $L(1, -2)$
- $D(-4, 3)$, $E(-2, 3)$, $F(-1, 1)$, $G(-4, 1)$ and $L(1, -1)$, $M(3, -1)$, $N(6, -3)$, $P(1, -3)$

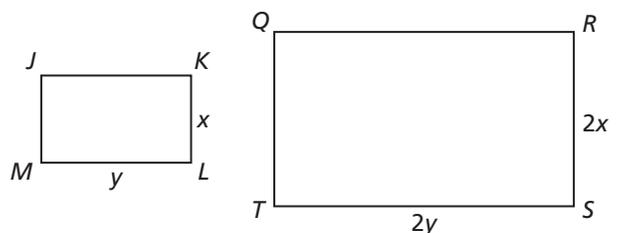
In Exercises 13 and 14, prove that the figures are similar. (See Example 3.)

- Given** Right isosceles $\triangle ABC$ with leg length j , right isosceles $\triangle RST$ with leg length k , $\overline{CA} \parallel \overline{RT}$

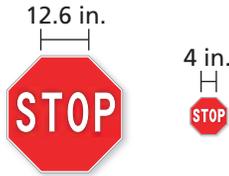
Prove $\triangle ABC$ is similar to $\triangle RST$.



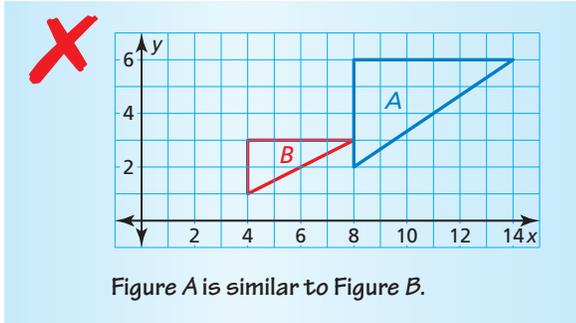
- Given** Rectangle $JKLM$ with side lengths x and y , rectangle $QRST$ with side lengths $2x$ and $2y$
Prove Rectangle $JKLM$ is similar to rectangle $QRST$.



15. **MODELING WITH MATHEMATICS** Determine whether the regular-sized stop sign and the stop sign sticker are similar. Use transformations to explain your reasoning.



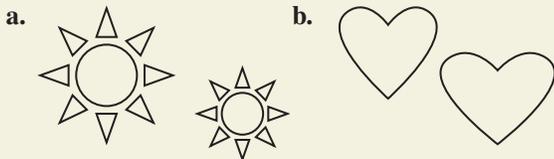
16. **ERROR ANALYSIS** Describe and correct the error in comparing the figures.



17. **MAKING AN ARGUMENT** A member of the homecoming decorating committee gives a printing company a banner that is 3 inches by 14 inches to enlarge. The committee member claims the banner she receives is distorted. Do you think the printing company distorted the image she gave it? Explain.



18. **HOW DO YOU SEE IT?** Determine whether each pair of figures is similar. Explain your reasoning.

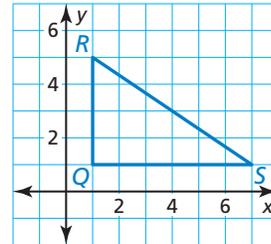


19. **ANALYZING RELATIONSHIPS** Graph a polygon in a coordinate plane. Use a similarity transformation involving a dilation (where k is a whole number) and a translation to graph a second polygon. Then describe a similarity transformation that maps the second polygon onto the first.

20. **THOUGHT PROVOKING** Is the composition of a rotation and a dilation commutative? (In other words, do you obtain the same image regardless of the order in which you perform the transformations?) Justify your answer.

21. **MATHEMATICAL CONNECTIONS** Quadrilateral $JKLM$ is mapped to quadrilateral $J'K'L'M'$ using the dilation $(x, y) \rightarrow (\frac{3}{2}x, \frac{3}{2}y)$. Then quadrilateral $J'K'L'M'$ is mapped to quadrilateral $J''K''L''M''$ using the translation $(x, y) \rightarrow (x + 3, y - 4)$. The vertices of quadrilateral $J'K'L'M'$ are $J'(-12, 0)$, $K'(-12, 18)$, $L'(-6, 18)$, and $M'(-6, 0)$. Find the coordinates of the vertices of quadrilateral $JKLM$ and quadrilateral $J''K''L''M''$. Are quadrilateral $JKLM$ and quadrilateral $J''K''L''M''$ similar? Explain.

22. **REPEATED REASONING** Use the diagram.



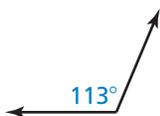
- a. Connect the midpoints of the sides of $\triangle QRS$ to make another triangle. Is this triangle similar to $\triangle QRS$? Use transformations to support your answer.
- b. Repeat part (a) for two other triangles. What conjecture can you make?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Classify the angle as *acute*, *obtuse*, *right*, or *straight*. (*Skills Review Handbook*)

23.



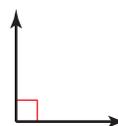
24.



25.



26.

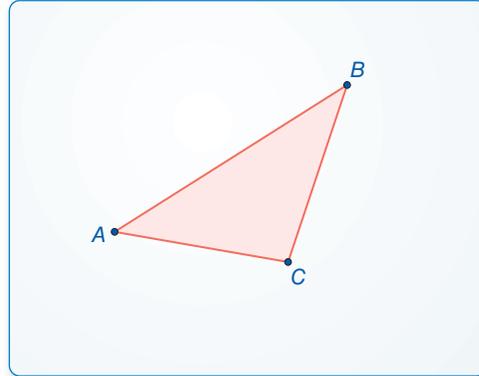


8.3 Similar Polygons

Essential Question How are similar polygons related?

EXPLORATION 1 Comparing Triangles after a Dilation

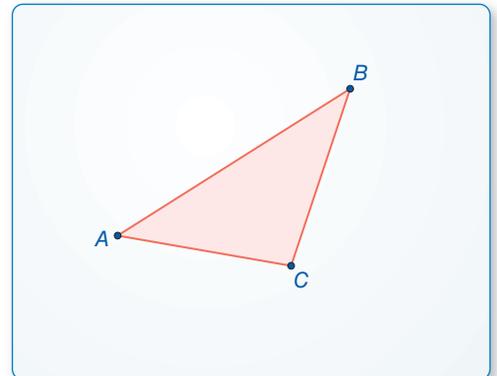
Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



- Compare the corresponding angles of $\triangle A'B'C'$ and $\triangle ABC$.
- Find the ratios of the lengths of the sides of $\triangle A'B'C'$ to the lengths of the corresponding sides of $\triangle ABC$. What do you observe?
- Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

EXPLORATION 2 Comparing Triangles after a Dilation

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



- Compare the perimeters of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?
- Compare the areas of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?
- Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

- How are similar polygons related?
- A $\triangle RST$ is dilated by a scale factor of 3 to form $\triangle R'S'T'$. The area of $\triangle RST$ is 1 square inch. What is the area of $\triangle R'S'T'$?

8.3 Lesson

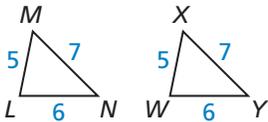
Core Vocabulary

Previous

similar figures
similarity transformation
corresponding parts

LOOKING FOR STRUCTURE

Notice that any two congruent figures are also similar. In $\triangle LMN$ and $\triangle WXY$ below, the scale factor is $\frac{5}{5} = \frac{6}{6} = \frac{7}{7} = 1$. So, you can write $\triangle LMN \sim \triangle WXY$ and $\triangle LMN \cong \triangle WXY$.



What You Will Learn

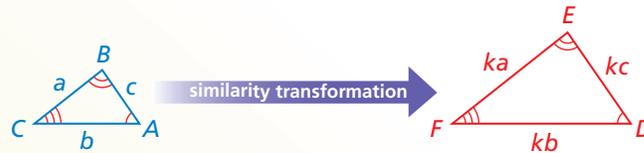
- ▶ Use similarity statements.
- ▶ Find corresponding lengths in similar polygons.
- ▶ Find perimeters and areas of similar polygons.
- ▶ Decide whether polygons are similar.

Using Similarity Statements

Core Concept

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write “ $\triangle ABC$ is similar to $\triangle DEF$ ” as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k . So, corresponding side lengths are proportional.



Corresponding angles

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

Ratios of corresponding side lengths

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

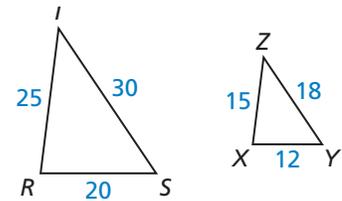
READING

In a *statement of proportionality*, any pair of ratios forms a true proportion.

EXAMPLE 1 Using Similarity Statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

- Find the scale factor from $\triangle RST$ to $\triangle XYZ$.
- List all pairs of congruent angles.
- Write the ratios of the corresponding side lengths in a *statement of proportionality*.



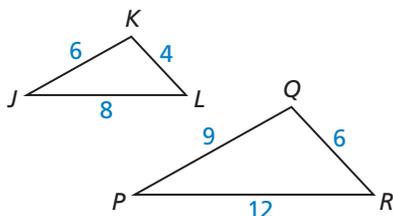
SOLUTION

$$\text{a. } \frac{XY}{RS} = \frac{12}{20} = \frac{3}{5} \qquad \frac{YZ}{ST} = \frac{18}{30} = \frac{3}{5} \qquad \frac{ZX}{TR} = \frac{15}{25} = \frac{3}{5}$$

So, the scale factor is $\frac{3}{5}$.

$$\text{b. } \angle R \cong \angle X, \angle S \cong \angle Y, \text{ and } \angle T \cong \angle Z.$$

$$\text{c. Because the ratios in part (a) are equal, } \frac{XY}{RS} = \frac{YZ}{ST} = \frac{ZX}{TR}.$$



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- In the diagram, $\triangle JKL \sim \triangle PQR$. Find the scale factor from $\triangle JKL$ to $\triangle PQR$. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

Finding Corresponding Lengths in Similar Polygons

Core Concept

READING

Corresponding lengths in similar triangles include side lengths, altitudes, medians, and midsegments.

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 2 Finding a Corresponding Length

In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x .

SOLUTION

The triangles are similar, so the corresponding side lengths are proportional.

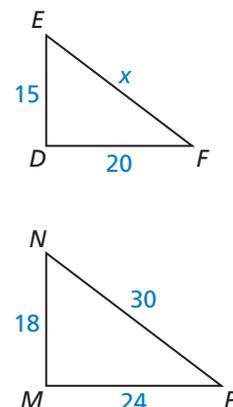
$$\frac{MN}{DE} = \frac{NP}{EF} \quad \text{Write proportion.}$$

$$\frac{18}{15} = \frac{30}{x} \quad \text{Substitute.}$$

$$18x = 450 \quad \text{Cross Products Property}$$

$$x = 25 \quad \text{Solve for } x.$$

▶ The value of x is 25.



FINDING AN ENTRY POINT

There are several ways to write the proportion. For example, you could write

$$\frac{DF}{MP} = \frac{EF}{NP}$$

EXAMPLE 3 Finding a Corresponding Length

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude \overline{PS} .

SOLUTION

First, find the scale factor from $\triangle XPZ$ to $\triangle TPR$.

$$\frac{TR}{XZ} = \frac{6 + 6}{8 + 8} = \frac{12}{16} = \frac{3}{4}$$

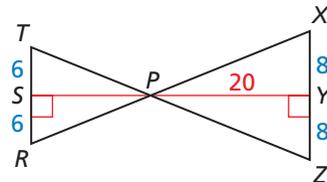
Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$$\frac{PS}{PY} = \frac{3}{4} \quad \text{Write proportion.}$$

$$\frac{PS}{20} = \frac{3}{4} \quad \text{Substitute 20 for } PY.$$

$$PS = 15 \quad \text{Multiply each side by 20 and simplify.}$$

▶ The length of the altitude \overline{PS} is 15.

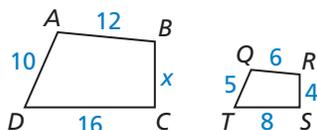


Monitoring Progress



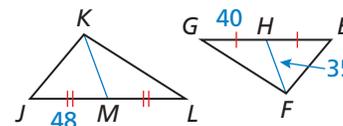
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2. Find the value of x .



$$ABCD \sim QRST$$

3. Find KM .



$$\triangle JKL \sim \triangle EFG$$

Finding Perimeters and Areas of Similar Polygons

Theorem

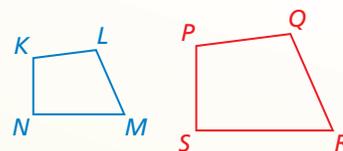
ANALYZING RELATIONSHIPS

When two similar polygons have a scale factor of k , the ratio of their perimeters is equal to k .



Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



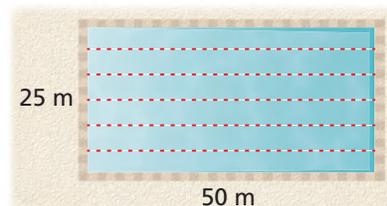
$$\text{If } KLMN \sim PQRS, \text{ then } \frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}.$$

Proof Ex. 52, p. 484; *BigIdeasMath.com*

EXAMPLE 4 Modeling with Mathematics



A town plans to build a new swimming pool. An Olympic pool is rectangular with a length of 50 meters and a width of 25 meters. The new pool will be similar in shape to an Olympic pool but will have a length of 40 meters. Find the perimeters of an Olympic pool and the new pool.



SOLUTION

- Understand the Problem** You are given the length and width of a rectangle and the length of a similar rectangle. You need to find the perimeters of both rectangles.
- Make a Plan** Find the scale factor of the similar rectangles and find the perimeter of an Olympic pool. Then use the Perimeters of Similar Polygons Theorem to write and solve a proportion to find the perimeter of the new pool.
- Solve the Problem** Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. Write and solve a proportion to find the perimeter x of the new pool.

$$\frac{x}{150} = \frac{4}{5}$$

Perimeters of Similar Polygons Theorem

$$x = 120$$

Multiply each side by 150 and simplify.

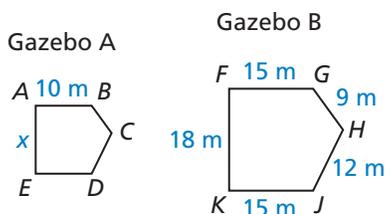
- So, the perimeter of an Olympic pool is 150 meters, and the perimeter of the new pool is 120 meters.

- Look Back** Check that the ratio of the perimeters is equal to the scale factor.

$$\frac{120}{150} = \frac{4}{5} \quad \checkmark$$

STUDY TIP

You can also write the scale factor as a decimal. In Example 4, you can write the scale factor as 0.8 and multiply by 150 to get $x = 0.8(150) = 120$.



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- The two gazebos shown are similar pentagons. Find the perimeter of Gazebo A.

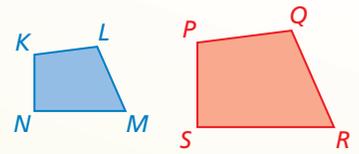
Theorem

ANALYZING RELATIONSHIPS

When two similar polygons have a scale factor of k , the ratio of their areas is equal to k^2 .

Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.

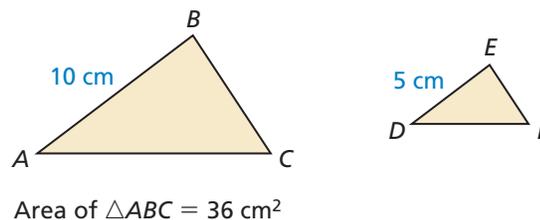


$$\text{If } KLMN \sim PQRS, \text{ then } \frac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\frac{PQ}{KL}\right)^2 = \left(\frac{QR}{LM}\right)^2 = \left(\frac{RS}{MN}\right)^2 = \left(\frac{SP}{NK}\right)^2.$$

Proof Ex. 53, p. 484; *BigIdeasMath.com*

EXAMPLE 5 Finding Areas of Similar Polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle DEF$.



SOLUTION

Because the triangles are similar, the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$ is equal to the square of the ratio of AB to DE . Write and solve a proportion to find the area of $\triangle DEF$. Let A represent the area of $\triangle DEF$.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2$$

Areas of Similar Polygons Theorem

$$\frac{36}{A} = \left(\frac{10}{5}\right)^2$$

Substitute.

$$\frac{36}{A} = \frac{100}{25}$$

Square the right side of the equation.

$$36 \cdot 25 = 100 \cdot A$$

Cross Products Property

$$900 = 100A$$

Simplify.

$$9 = A$$

Solve for A .

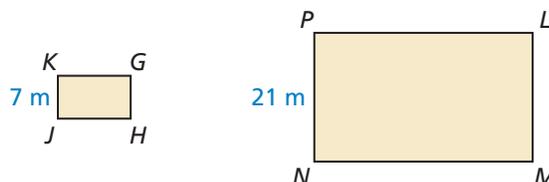
► The area of $\triangle DEF$ is 9 square centimeters.

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5. In the diagram, $GHJK \sim LMNP$. Find the area of $LMNP$.

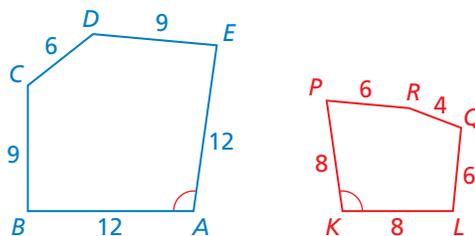


Area of $GHJK = 84 \text{ m}^2$

Deciding Whether Polygons Are Similar

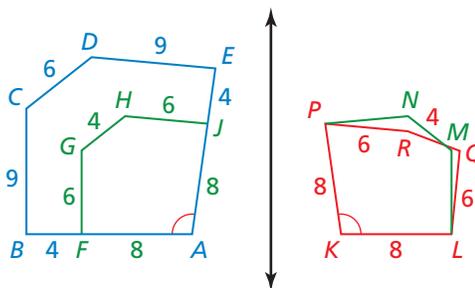
EXAMPLE 6 Deciding Whether Polygons Are Similar

Decide whether $ABCDE$ and $KLQRP$ are similar. Explain your reasoning.



SOLUTION

Corresponding sides of the pentagons are proportional with a scale factor of $\frac{2}{3}$. However, this does not necessarily mean the pentagons are similar. A dilation with center A and scale factor $\frac{2}{3}$ moves $ABCDE$ onto $AFGHJ$. Then a reflection moves $AFGHJ$ onto $KLMNP$.

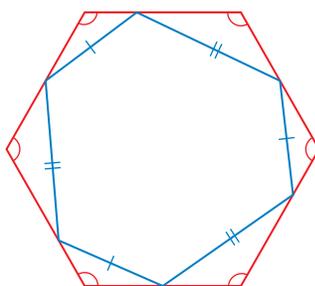


$KLMNP$ does not exactly coincide with $KLQRP$, because not all the corresponding angles are congruent. (Only $\angle A$ and $\angle K$ are congruent.)

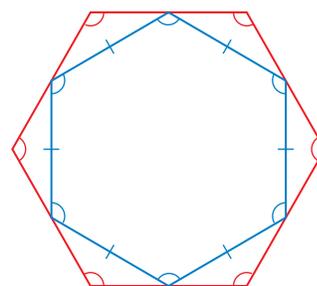
► Because angle measure is not preserved, the two pentagons are not similar.

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Refer to the floor tile designs below. In each design, the red shape is a regular hexagon.



Tile Design 1



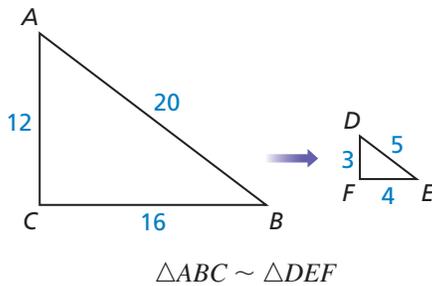
Tile Design 2

- Decide whether the hexagons in Tile Design 1 are similar. Explain.
- Decide whether the hexagons in Tile Design 2 are similar. Explain.

8.3 Exercises

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** For two figures to be similar, the corresponding angles must be _____, and the corresponding side lengths must be _____.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.



What is the scale factor?

What is the ratio of their areas?

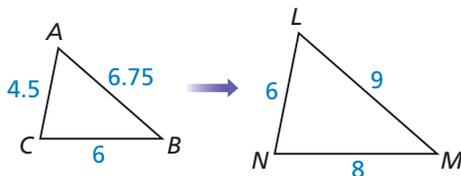
What is the ratio of their corresponding side lengths?

What is the ratio of their perimeters?

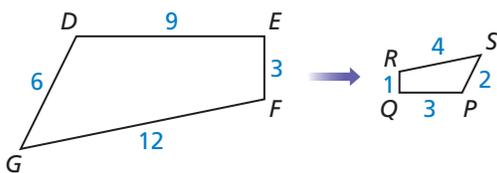
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality. (See Example 1.)

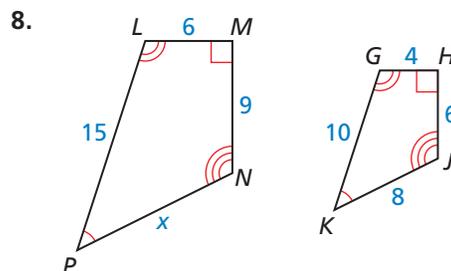
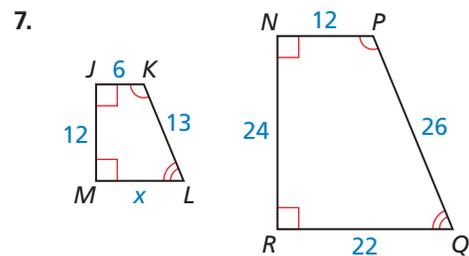
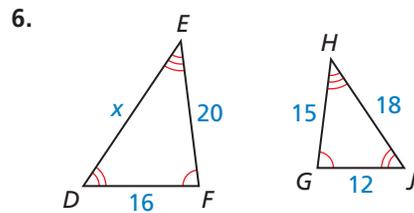
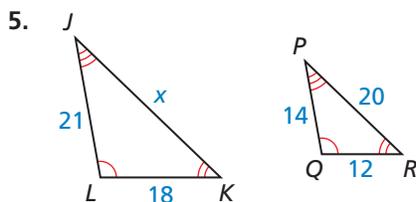
3. $\triangle ABC \sim \triangle LMN$



4. $DEFG \sim PQRS$

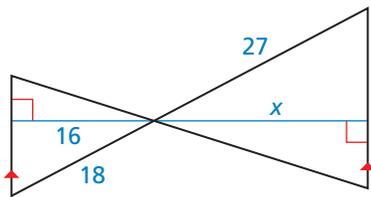


In Exercises 5–8, the polygons are similar. Find the value of x . (See Example 2.)

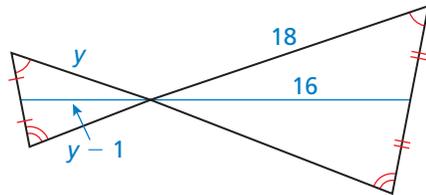


In Exercises 9 and 10, the black triangles are similar. Identify the type of segment shown in blue and find the value of the variable. (See Example 3.)

9.

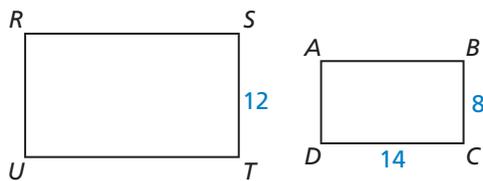


10.

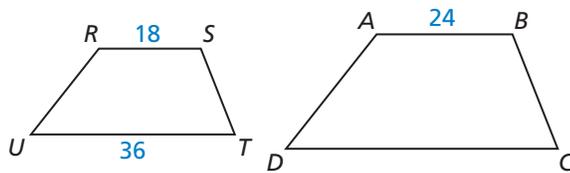


In Exercises 11 and 12, $RSTU \sim ABCD$. Find the ratio of their perimeters.

11.



12.



In Exercises 13–16, two polygons are similar. The perimeter of one polygon and the ratio of the corresponding side lengths are given. Find the perimeter of the other polygon.

13. perimeter of smaller polygon: 48 cm; ratio: $\frac{2}{3}$

14. perimeter of smaller polygon: 66 ft; ratio: $\frac{3}{4}$

15. perimeter of larger polygon: 120 yd; ratio: $\frac{1}{6}$

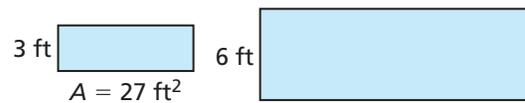
16. perimeter of larger polygon: 85 m; ratio: $\frac{2}{5}$

17. **MODELING WITH MATHEMATICS** A school gymnasium is being remodeled. The basketball court will be similar to an NCAA basketball court, which has a length of 94 feet and a width of 50 feet. The school plans to make the width of the new court 45 feet. Find the perimeters of an NCAA court and of the new court in the school. (See Example 4.)

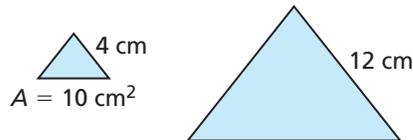
18. **MODELING WITH MATHEMATICS** Your family has decided to put a rectangular patio in your backyard, similar to the shape of your backyard. Your backyard has a length of 45 feet and a width of 20 feet. The length of your new patio is 18 feet. Find the perimeters of your backyard and of the patio.

In Exercises 19–22, the polygons are similar. The area of one polygon is given. Find the area of the other polygon. (See Example 5.)

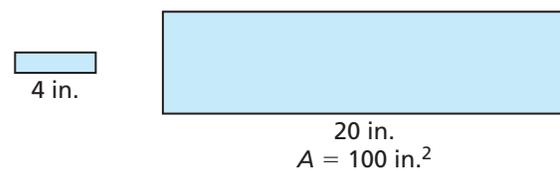
19.



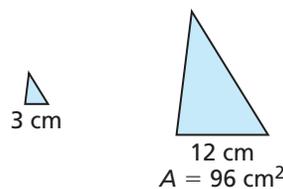
20.



21.



22.



23. **ERROR ANALYSIS** Describe and correct the error in finding the perimeter of triangle B. The triangles are similar.

X

$$\frac{5}{10} = \frac{28}{x}$$

$$5x = 280$$

$$x = 56$$

24. **ERROR ANALYSIS** Describe and correct the error in finding the area of rectangle B. The rectangles are similar.

X

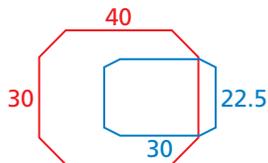
$$\frac{6}{18} = \frac{24}{x}$$

$$6x = 432$$

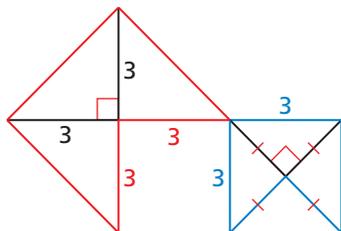
$$x = 72$$

In Exercises 25 and 26, decide whether the red and blue polygons are similar. (See Example 6.)

25.



26.



27. **REASONING** Triangles ABC and DEF are similar. Which statement is correct? Select all that apply.

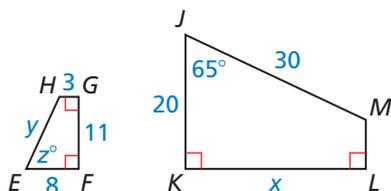
(A) $\frac{BC}{EF} = \frac{AC}{DF}$

(B) $\frac{AB}{DE} = \frac{CA}{FE}$

(C) $\frac{AB}{EF} = \frac{BC}{DE}$

(D) $\frac{CA}{FD} = \frac{BC}{EF}$

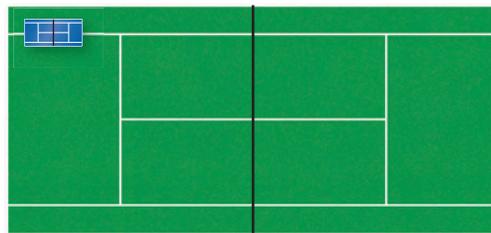
ANALYZING RELATIONSHIPS In Exercises 28–34, $JKLM \sim EFGH$.



28. Find the scale factor of $JKLM$ to $EFGH$.
29. Find the scale factor of $EFGH$ to $JKLM$.
30. Find the values of x , y , and z .
31. Find the perimeter of each polygon.
32. Find the ratio of the perimeters of $JKLM$ to $EFGH$.
33. Find the area of each polygon.
34. Find the ratio of the areas of $JKLM$ to $EFGH$.
35. **USING STRUCTURE** Rectangle A is similar to rectangle B. Rectangle A has side lengths of 6 and 12. Rectangle B has a side length of 18. What are the possible values for the length of the other side of rectangle B? Select all that apply.

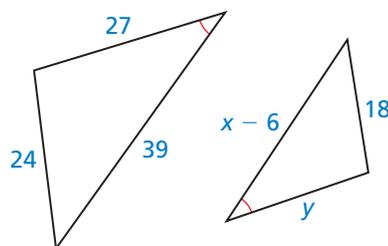
(A) 6 (B) 9 (C) 24 (D) 36

36. **DRAWING CONCLUSIONS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? Explain. If so, find the scale factor of the tennis court to the table.

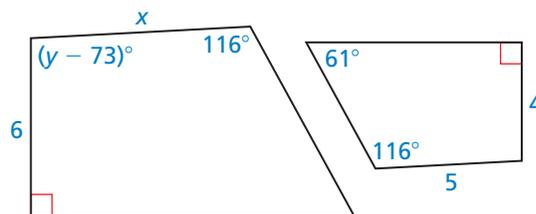


MATHEMATICAL CONNECTIONS In Exercises 37 and 38, the two polygons are similar. Find the values of x and y .

37.



38.



ATTENDING TO PRECISION In Exercises 39–42, the figures are similar. Find the missing corresponding side length.

39. Figure A has a perimeter of 72 meters and one of the side lengths is 18 meters. Figure B has a perimeter of 120 meters.
40. Figure A has a perimeter of 24 inches. Figure B has a perimeter of 36 inches and one of the side lengths is 12 inches.
41. Figure A has an area of 48 square feet and one of the side lengths is 6 feet. Figure B has an area of 75 square feet.
42. Figure A has an area of 18 square feet. Figure B has an area of 98 square feet and one of the side lengths is 14 feet.

CRITICAL THINKING In Exercises 43–48, tell whether the polygons are *always*, *sometimes*, or *never* similar.

43. two isosceles triangles 44. two isosceles trapezoids

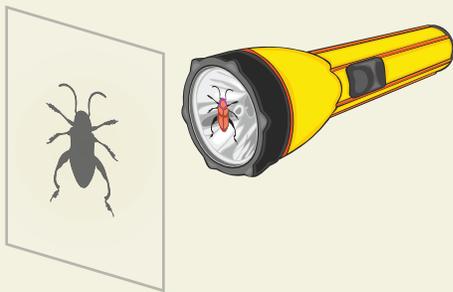
45. two rhombuses 46. two squares

47. two regular polygons

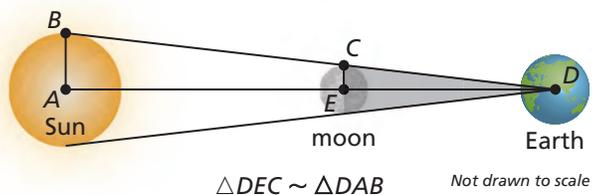
48. a right triangle and an equilateral triangle

49. **MAKING AN ARGUMENT** Your sister claims that when the side lengths of two rectangles are proportional, the two rectangles must be similar. Is she correct? Explain your reasoning.

50. **HOW DO YOU SEE IT?** You shine a flashlight directly on an object to project its image onto a parallel screen. Will the object and the image be similar? Explain your reasoning.



51. **MODELING WITH MATHEMATICS** During a total eclipse of the Sun, the moon is directly in line with the Sun and blocks the Sun's rays. The distance DA between Earth and the Sun is 93,000,000 miles, the distance DE between Earth and the moon is 240,000 miles, and the radius AB of the Sun is 432,500 miles. Use the diagram and the given measurements to estimate the radius EC of the moon.

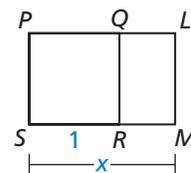


52. **PROVING A THEOREM** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

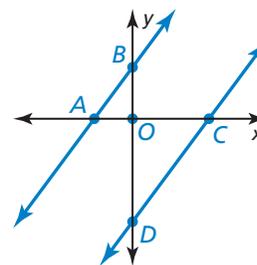
53. **PROVING A THEOREM** Prove the Areas of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

54. **THOUGHT PROVOKING** The postulates and theorems you have learned represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. In spherical geometry, is it possible that two triangles are similar but not congruent? Explain your reasoning.

55. **CRITICAL THINKING** In the diagram, $PQRS$ is a square, and $PLMS \sim LMRQ$. Find the exact value of x . This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



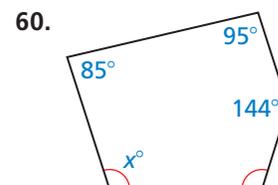
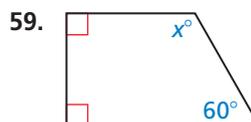
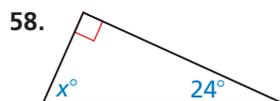
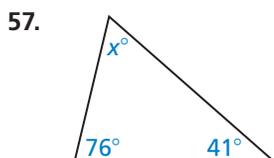
56. **MATHEMATICAL CONNECTIONS** The equations of the lines shown are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$. Show that $\triangle AOB \sim \triangle COD$.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of x . (Section 7.1)



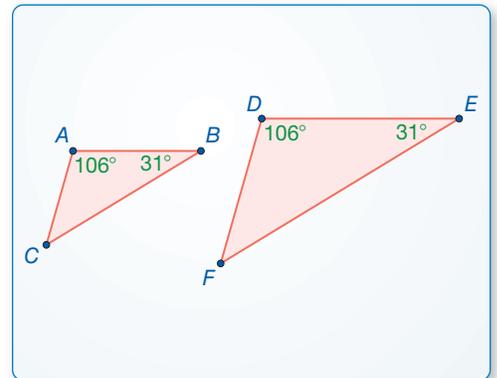
8.4 Proving Triangle Similarity by AA

Essential Question What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?

EXPLORATION 1 Comparing Triangles

Work with a partner. Use dynamic geometry software.

- a. Construct $\triangle ABC$ and $\triangle DEF$ so that $m\angle A = m\angle D = 106^\circ$, $m\angle B = m\angle E = 31^\circ$, and $\triangle DEF$ is not congruent to $\triangle ABC$.



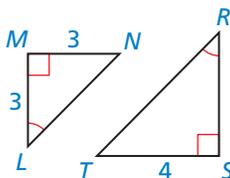
- b. Find the third angle measure and the side lengths of each triangle. Copy the table below and record your results in column 1.

	1.	2.	3.	4.	5.	6.
$m\angle A, m\angle D$	106°	88°	40°			
$m\angle B, m\angle E$	31°	42°	65°			
$m\angle C$						
$m\angle F$						
AB						
DE						
BC						
EF						
AC						
DF						

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in constructing arguments.

- c. Are the two triangles similar? Explain.
- d. Repeat parts (a)–(c) to complete columns 2 and 3 of the table for the given angle measures.
- e. Complete each remaining column of the table using your own choice of two pairs of equal corresponding angle measures. Can you construct two triangles in this way that are *not* similar?
- f. Make a conjecture about any two triangles with two pairs of congruent corresponding angles.



Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?
3. Find RS in the figure at the left.

8.4 Lesson

Core Vocabulary

Previous
similar figures
similarity transformation

What You Will Learn

- ▶ Use the Angle-Angle Similarity Theorem.
- ▶ Solve real-life problems.

Using the Angle-Angle Similarity Theorem

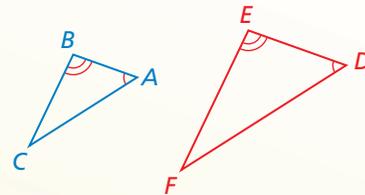
Theorem

Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,
then $\triangle ABC \sim \triangle DEF$.

Proof p. 486

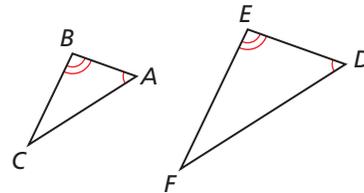


PROOF

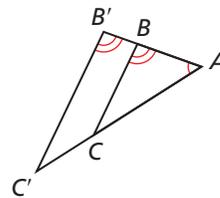
Angle-Angle (AA) Similarity Theorem

Given $\angle A \cong \angle D$, $\angle B \cong \angle E$

Prove $\triangle ABC \sim \triangle DEF$



Dilate $\triangle ABC$ using a scale factor of $k = \frac{DE}{AB}$ and center A . The image of $\triangle ABC$ is $\triangle AB'C'$.



Because a dilation is a similarity transformation, $\triangle ABC \sim \triangle AB'C'$. Because the ratio of corresponding lengths of similar polygons equals the scale factor, $\frac{AB'}{AB} = \frac{DE}{AB}$.

Multiplying each side by AB yields $AB' = DE$. By the definition of congruent segments, $\overline{AB'} \cong \overline{DE}$.

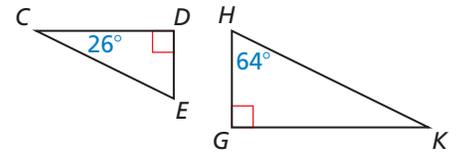
By the Reflexive Property of Congruence, $\angle A \cong \angle A$. Because corresponding angles of similar polygons are congruent, $\angle B' \cong \angle B$. Because $\angle B' \cong \angle B$ and $\angle B \cong \angle E$, $\angle B' \cong \angle E$ by the Transitive Property of Congruence.

Because $\angle A \cong \angle D$, $\angle B' \cong \angle E$, and $\overline{AB'} \cong \overline{DE}$, $\triangle AB'C' \cong \triangle DEF$ by the ASA Congruence Theorem. So, a composition of rigid motions maps $\triangle AB'C'$ to $\triangle DEF$.

Because a dilation followed by a composition of rigid motions maps $\triangle ABC$ to $\triangle DEF$, $\triangle ABC \sim \triangle DEF$.

EXAMPLE 1 Using the AA Similarity Theorem

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



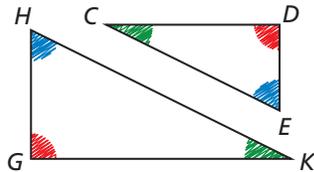
SOLUTION

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

By the Triangle Sum Theorem, $26^\circ + 90^\circ + m\angle E = 180^\circ$, so $m\angle E = 64^\circ$. So, $\angle E$ and $\angle H$ are congruent.

► So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Theorem.

VISUAL REASONING

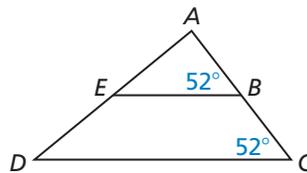


Use colored pencils to show congruent angles. This will help you write similarity statements.

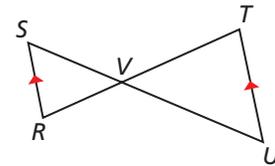
EXAMPLE 2 Using the AA Similarity Theorem

Show that the two triangles are similar.

a. $\triangle ABE \sim \triangle ACD$



b. $\triangle SVR \sim \triangle UVT$

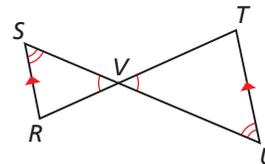


SOLUTION

a. Because $m\angle ABE$ and $m\angle C$ both equal 52° , $\angle ABE \cong \angle C$. By the Reflexive Property of Congruence, $\angle A \cong \angle A$.

► So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Theorem.

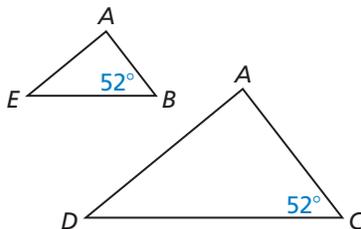
b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$, so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.



► So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Theorem.

VISUAL REASONING

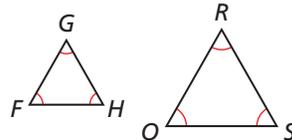
You may find it helpful to redraw the triangles separately.



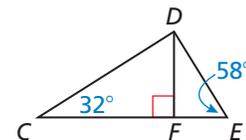
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Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. **WHAT IF?** Suppose that $\overline{SR} \not\parallel \overline{TU}$ in Example 2 part (b). Could the triangles still be similar? Explain.

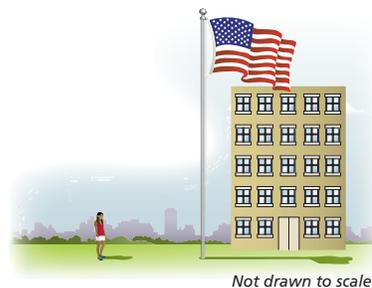
Solving Real-Life Problems

Previously, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Modeling with Mathematics

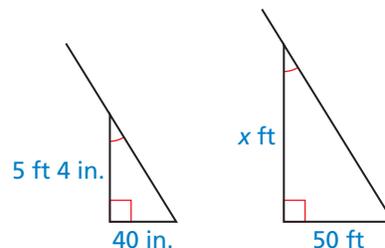


A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is 5 feet 4 inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?



SOLUTION

- Understand the Problem** You are given the length of a flagpole's shadow, the height of a woman, and the length of the woman's shadow. You need to find the height of the flagpole.
- Make a Plan** Use similar triangles to write a proportion and solve for the height of the flagpole.
- Solve the Problem** The flagpole and the woman form sides of two right triangles with the ground. The Sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Theorem.



You can use a proportion to find the height x . Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}} \quad \text{Write proportion of side lengths.}$$

$$40x = 3200 \quad \text{Cross Products Property}$$

$$x = 80 \quad \text{Solve for } x.$$

► The flagpole is 80 feet tall.

- Look Back** Attend to precision by checking that your answer has the correct units. The problem asks for the height of the flagpole to the nearest *foot*. Because your answer is 80 feet, the units match.

Also, check that your answer is reasonable in the context of the problem. A height of 80 feet makes sense for a flagpole. You can estimate that an eight-story building would be about $8(10 \text{ feet}) = 80$ feet, so it is reasonable that a flagpole could be that tall.

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- WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?
- You are standing outside, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

8.4 Exercises

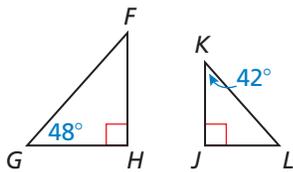
Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are _____.
- WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? Explain.

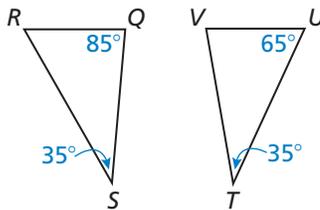
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning. (See Example 1.)

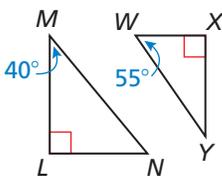
3.



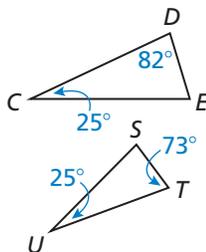
4.



5.

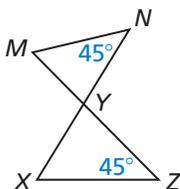


6.

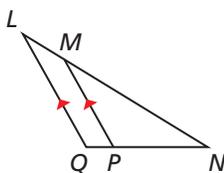


In Exercises 7–10, show that the two triangles are similar. (See Example 2.)

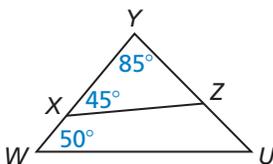
7.



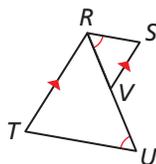
8.



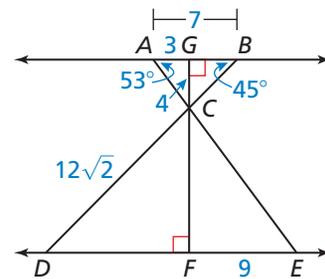
9.



10.



In Exercises 11–18, use the diagram to copy and complete the statement.



- | | |
|---|---|
| 11. $\triangle CAG \sim$ <input type="text"/> | 12. $\triangle DCF \sim$ <input type="text"/> |
| 13. $\triangle ACB \sim$ <input type="text"/> | 14. $m\angle ECF =$ <input type="text"/> |
| 15. $m\angle ECD =$ <input type="text"/> | 16. $CF =$ <input type="text"/> |
| 17. $BC =$ <input type="text"/> | 18. $DE =$ <input type="text"/> |

19. **ERROR ANALYSIS** Describe and correct the error in using the AA Similarity Theorem.

X

Quadrilateral $ABCD \sim$ quadrilateral $EFGH$ by the AA Similarity Theorem.

20. **ERROR ANALYSIS** Describe and correct the error in finding the value of x .

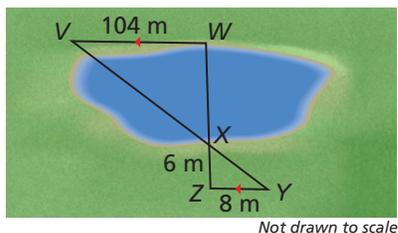
X

$$\frac{4}{6} = \frac{5}{x}$$

$$4x = 30$$

$$x = 7.5$$

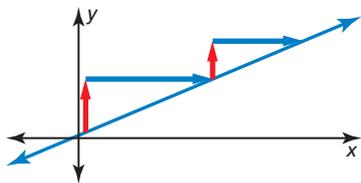
21. **MODELING WITH MATHEMATICS** You can measure the width of the lake using a surveying technique, as shown in the diagram. Find the width of the lake, WX . Justify your answer. (See Example 3.)



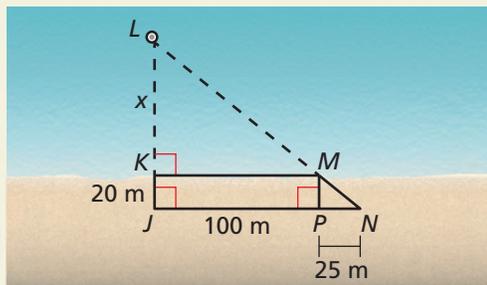
22. **MAKING AN ARGUMENT** You and your cousin are trying to determine the height of a telephone pole. Your cousin tells you to stand in the pole's shadow so that the tip of your shadow coincides with the tip of the pole's shadow. Your cousin claims to be able to use the distance between the tips of the shadows and you, the distance between you and the pole, and your height to estimate the height of the telephone pole. Is this possible? Explain. Include a diagram in your answer.

REASONING In Exercises 23–26, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? Explain your reasoning.

23. $m\angle J = 71^\circ$, $m\angle K = 52^\circ$, $m\angle X = 71^\circ$, and $m\angle Z = 57^\circ$
24. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150^\circ$.
25. $m\angle L = 87^\circ$ and $m\angle Y = 94^\circ$
26. $m\angle J + m\angle K = 85^\circ$ and $m\angle Y + m\angle Z = 80^\circ$
27. **MATHEMATICAL CONNECTIONS** Explain how you can use similar triangles to show that any two points on a line can be used to find its slope.



28. **HOW DO YOU SEE IT?** In the diagram, which triangles would you use to find the distance x between the shoreline and the buoy? Explain your reasoning.

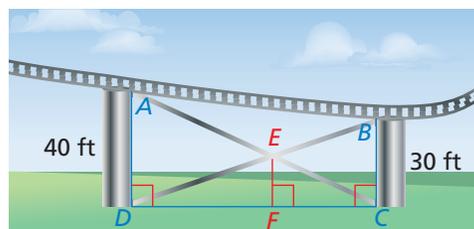


29. **WRITING** Explain why all equilateral triangles are similar.

30. **THOUGHT PROVOKING** Decide whether each is a valid method of showing that two quadrilaterals are similar. Justify your answer.

- a. AAA b. AAAA

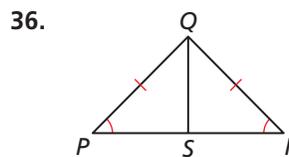
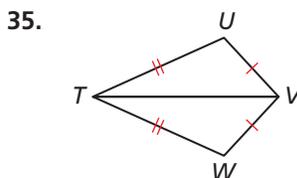
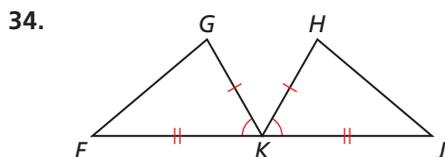
31. **PROOF** Without using corresponding lengths in similar polygons, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.
32. **PROOF** Prove that if the lengths of two sides of a triangle are a and b , respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.
33. **MODELING WITH MATHEMATICS** A portion of an amusement park ride is shown. Find EF . Justify your answer.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether there is enough information to prove that the triangles are congruent. Explain your reasoning. (Skills Review Handbook)



8.1–8.4 What Did You Learn?

Core Vocabulary

dilation, *p. 462*
center of dilation, *p. 462*
scale factor, *p. 462*

enlargement, *p. 462*
reduction, *p. 462*
similarity transformation, *p. 470*

similar figures, *p. 470*

Core Concepts

Section 8.1

Dilations and Scale Factor, *p. 462*
Coordinate Rule for Dilations, *p. 463*

Negative Scale Factors, *p. 464*

Section 8.2

Similarity Transformations, *p. 470*

Section 8.3

Corresponding Parts of Similar Polygons, *p. 476*
Corresponding Lengths in Similar Polygons, *p. 477*

Perimeters of Similar Polygons, *p. 478*
Areas of Similar Polygons, *p. 479*

Section 8.4

Angle-Angle (AA) Similarity Theorem, *p. 486*

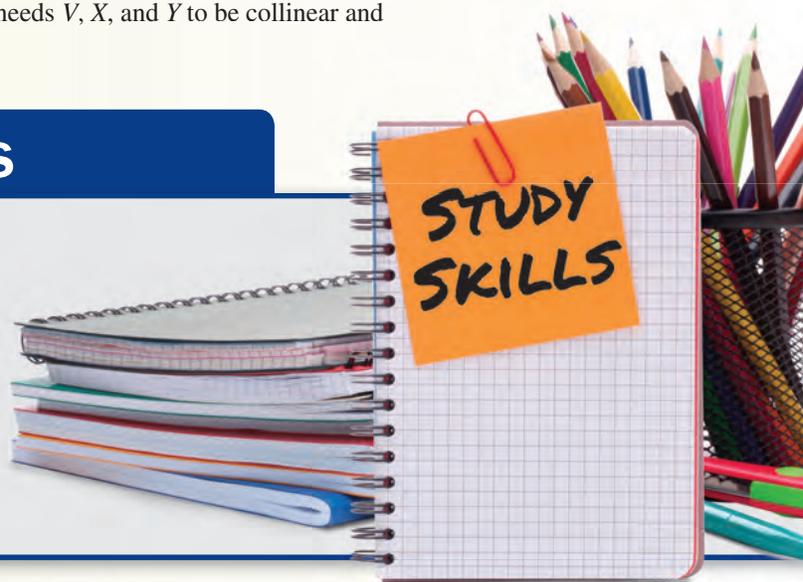
Mathematical Practices

1. In Exercise 35 on page 483, why is there more than one correct answer for the length of the other side?
2. In Exercise 50 on page 484, how could you find the scale factor of the similar figures? Describe any tools that might be helpful.
3. In Exercise 21 on page 490, explain why the surveyor needs V , X , and Y to be collinear and Z , X , and W to be collinear.

Analyzing Your Errors

Misreading Directions

- **What Happens:** You incorrectly read or do not understand directions.
- **How to Avoid This Error:** Read the instructions for exercises at least twice and make sure you understand what they mean. Make this a habit and use it when taking tests.



8.1–8.4 Quiz

Graph the triangle and its image after a dilation with scale factor k . (Section 8.1)

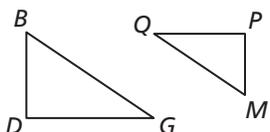
- $A(3, 6), B(3, 9), C(6, 3); k = \frac{2}{3}$
- $D(-2, -1), E(3, 1), F(1, -3); k = -2$
- $M(-4, -2), N(8, 6), P(4, -4); k = 0.5$
- $Q(-3, -2), R(-4, 3), S(2, -4); k = 300\%$

Describe a similarity transformation that maps $\triangle JKL$ to $\triangle MNP$. (Section 8.2)

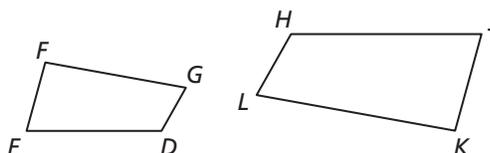
- $J(2, 1), K(-3, -2), L(1, -3)$ and $M(6, 4), N(-4, -2), P(4, -4)$
- $J(6, 2), K(-4, 2), L(-6, 4)$ and $M(-3, 1), N(2, 1), P(3, 2)$

List all pairs of congruent angles. Then write the ratios of the corresponding side lengths in a statement of proportionality. (Section 8.3)

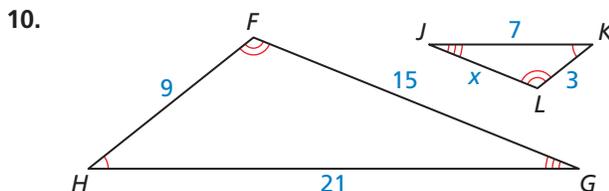
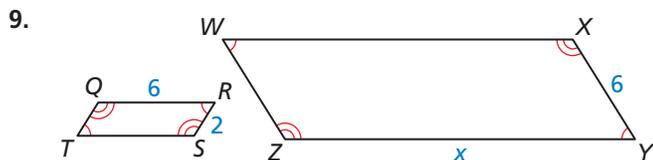
7. $\triangle BDG \sim \triangle MPQ$



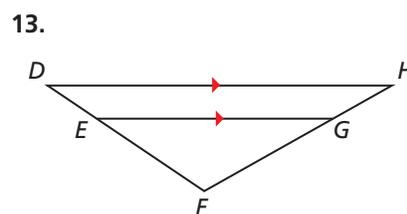
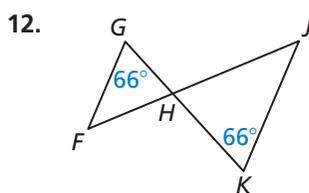
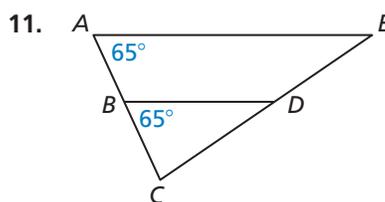
8. $DEFG \sim HJKL$



The polygons are similar. Find the value of x . (Section 8.3)

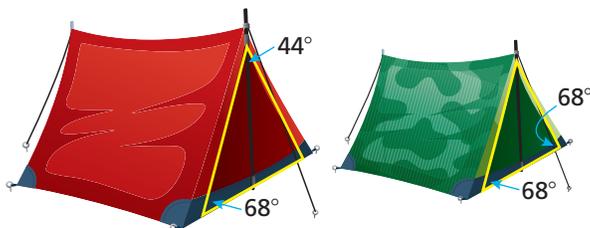


Show that the two triangles are similar. (Section 8.4)



14. The dimensions of an official hockey rink used by the National Hockey League (NHL) are 200 feet by 85 feet. The dimensions of an air hockey table are 96 inches by 40.8 inches. Assume corresponding angles are congruent. (Section 8.3)

- Determine whether the two surfaces are similar.
- If the surfaces are similar, find the ratio of their perimeters and the ratio of their areas. If not, find the dimensions of an air hockey table that are similar to an NHL hockey rink.



15. You and a friend buy camping tents made by the same company but in different sizes and colors. Use the information given in the diagram to decide whether the triangular faces of the tents are similar. Explain your reasoning. (Section 8.4)

8.5 Proving Triangle Similarity by SSS and SAS

Essential Question What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

EXPLORATION 1 Deciding Whether Triangles Are Similar

Work with a partner. Use dynamic geometry software.

- a. Construct $\triangle ABC$ and $\triangle DEF$ with the side lengths given in column 1 of the table below.

	1.	2.	3.	4.	5.	6.	7.
AB	5	5	6	15	9	24	
BC	8	8	8	20	12	18	
AC	10	10	10	10	8	16	
DE	10	15	9	12	12	8	
EF	16	24	12	16	15	6	
DF	20	30	15	8	10	8	
$m\angle A$							
$m\angle B$							
$m\angle C$							
$m\angle D$							
$m\angle E$							
$m\angle F$							

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to analyze situations by breaking them into cases and recognize and use counterexamples.

- b. Copy the table and complete column 1.
- c. Are the triangles similar? Explain your reasoning.
- d. Repeat parts (a)–(c) for columns 2–6 in the table.
- e. How are the corresponding side lengths related in each pair of triangles that are similar? Is this true for each pair of triangles that are not similar?
- f. Make a conjecture about the similarity of two triangles based on their corresponding side lengths.
- g. Use your conjecture to write another set of side lengths of two similar triangles. Use the side lengths to complete column 7 of the table.

EXPLORATION 2 Deciding Whether Triangles Are Similar

Work with a partner. Use dynamic geometry software. Construct any $\triangle ABC$.

- a. Find AB , AC , and $m\angle A$. Choose any positive rational number k and construct $\triangle DEF$ so that $DE = k \cdot AB$, $DF = k \cdot AC$, and $m\angle D = m\angle A$.
- b. Is $\triangle DEF$ similar to $\triangle ABC$? Explain your reasoning.
- c. Repeat parts (a) and (b) several times by changing $\triangle ABC$ and k . Describe your results.

Communicate Your Answer

3. What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

8.5 Lesson

Core Vocabulary

Previous
similar figures
corresponding parts
parallel lines

What You Will Learn

- ▶ Use the Side-Side-Side Similarity Theorem.
- ▶ Use the Side-Angle-Side Similarity Theorem.

Using the Side-Side-Side Similarity Theorem

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

Theorem

Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

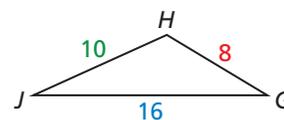
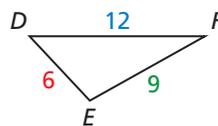
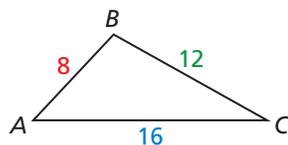


If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof p. 495

EXAMPLE 1 Using the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



FINDING AN ENTRY POINT

When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

SOLUTION

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides

$$\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$$

Longest sides

$$\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$$

Remaining sides

$$\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$$

- ▶ All the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

Shortest sides

$$\frac{AB}{GH} = \frac{8}{8} = 1$$

Longest sides

$$\frac{CA}{JG} = \frac{16}{16} = 1$$

Remaining sides

$$\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$$

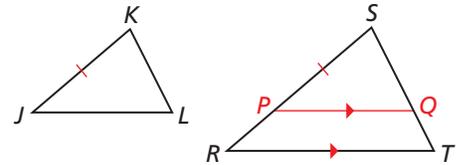
- ▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF

 SSS Similarity Theorem

Given $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

Prove $\triangle RST \sim \triangle JKL$



JUSTIFYING STEPS

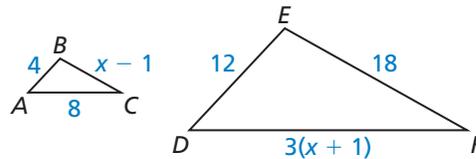
The Parallel Postulate allows you to draw an auxiliary line \overline{PQ} in $\triangle RST$. There is only one line through point P parallel to \overline{RT} , so you are able to draw it.

Locate P on \overline{RS} so that $PS = JK$. Draw \overline{PQ} so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Theorem, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$. You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Theorem, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Theorem to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2

 Using the SSS Similarity Theorem

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



FINDING AN ENTRY POINT

You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

SOLUTION

Step 1 Find the value of x that makes corresponding side lengths proportional.

$$\frac{AB}{DE} = \frac{BC}{EF}$$

Write proportion.

$$\frac{4}{12} = \frac{x - 1}{18}$$

Substitute.

$$4 \cdot 18 = 12(x - 1)$$

Cross Products Property

$$72 = 12x - 12$$

Simplify.

$$7 = x$$

Solve for x .

Step 2 Check that the side lengths are proportional when $x = 7$.

$$BC = x - 1 = 6$$

$$DF = 3(x + 1) = 24$$

$$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{12} = \frac{6}{18} \checkmark$$

$$\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \rightarrow \frac{4}{12} = \frac{8}{24} \checkmark$$

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

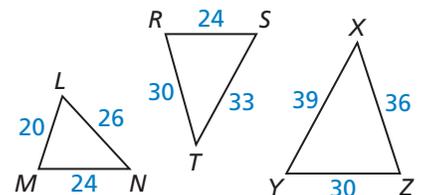
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Use the diagram.

- Which of the three triangles are similar? Write a similarity statement.
- The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.

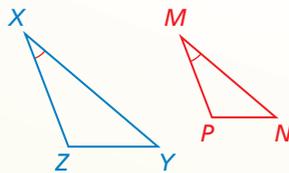


Using the Side-Angle-Side Similarity Theorem

Theorem

Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



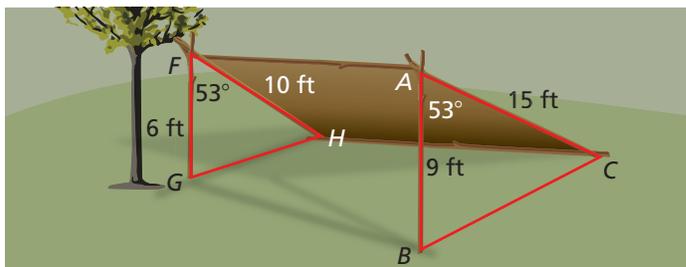
If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof Ex. 19, p. 498

EXAMPLE 3 Using the SAS Similarity Theorem



You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



SOLUTION

Both $m\angle A$ and $m\angle F$ equal 53° , so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides

$$\begin{aligned} \frac{AB}{FG} &= \frac{9}{6} \\ &= \frac{3}{2} \end{aligned}$$

Longer sides

$$\begin{aligned} \frac{AC}{FH} &= \frac{15}{10} \\ &= \frac{3}{2} \end{aligned}$$

The lengths of the sides that include $\angle A$ and $\angle F$ are proportional. So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$.

► Yes, you can make the right end similar to the left end of the shelter.

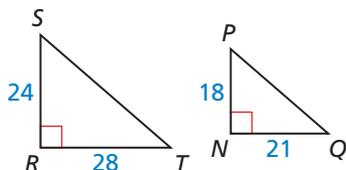
Monitoring Progress



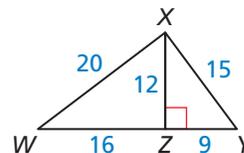
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Explain how to show that the indicated triangles are similar.

3. $\triangle SRT \sim \triangle PNQ$



4. $\triangle XZW \sim \triangle YZX$



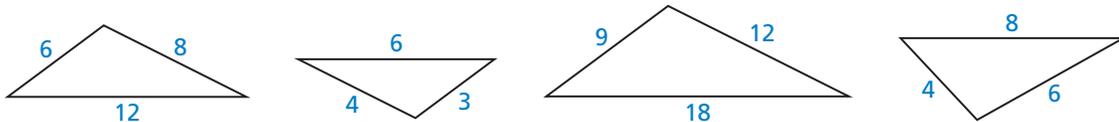
8.5 Exercises

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** You plan to show that $\triangle QRS$ is similar to $\triangle XYZ$ by the SSS Similarity Theorem.

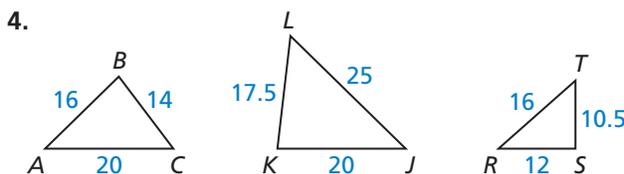
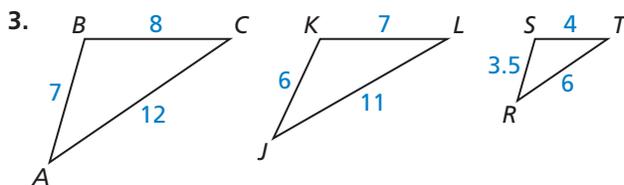
Copy and complete the proportion that you will use: $\frac{QR}{\square} = \frac{\square}{YZ} = \frac{QS}{\square}$.

2. **WHICH ONE DOESN'T BELONG?** Which triangle does *not* belong with the other three? Explain your reasoning.

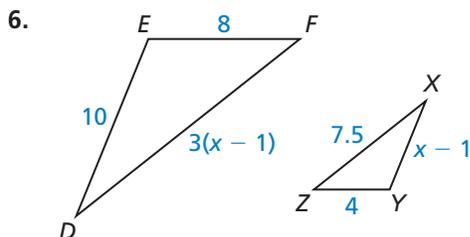
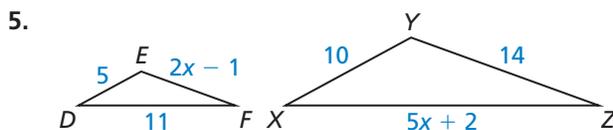


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In Exercises 3 and 4, determine whether $\triangle JKL$ or $\triangle RST$ is similar to $\triangle ABC$. (See Example 1.)



In Exercises 5 and 6, find the value of x that makes $\triangle DEF \sim \triangle XYZ$. (See Example 2.)

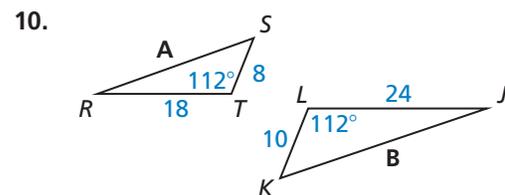
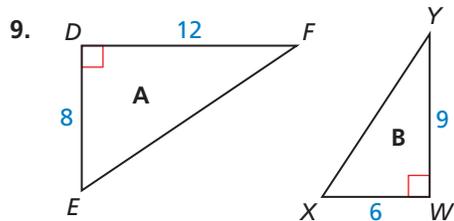


In Exercises 7 and 8, verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

7. $\triangle ABC$: $BC = 18, AB = 15, AC = 12$
 $\triangle DEF$: $EF = 12, DE = 10, DF = 8$

8. $\triangle ABC$: $AB = 10, BC = 16, CA = 20$
 $\triangle DEF$: $DE = 25, EF = 40, FD = 50$

In Exercises 9 and 10, determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of triangle B to triangle A. (See Example 3.)

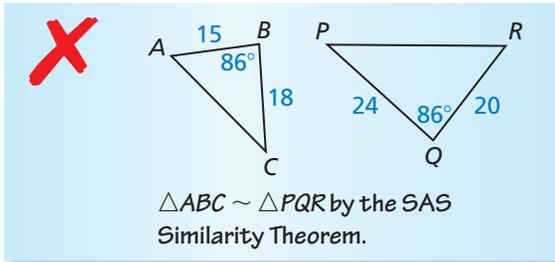


In Exercises 11 and 12, sketch the triangles using the given description. Then determine whether the two triangles can be similar.

11. In $\triangle RST$, $RS = 20, ST = 32$, and $m\angle S = 16^\circ$. In $\triangle FGH$, $GH = 30, HF = 48$, and $m\angle H = 24^\circ$.

12. The side lengths of $\triangle ABC$ are 24, $8x$, and 48, and the side lengths of $\triangle DEF$ are 15, 25, and $6x$.

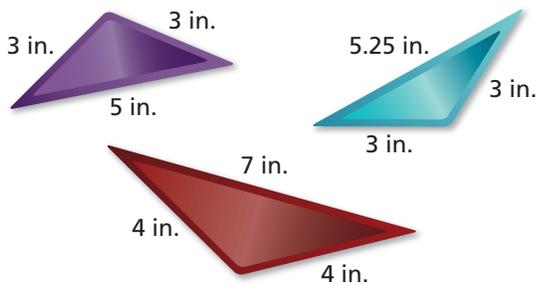
13. **ERROR ANALYSIS** Describe and correct the error in writing a similarity statement.



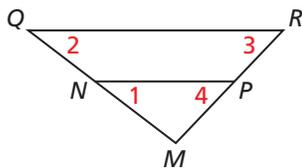
14. **MATHEMATICAL CONNECTIONS** Find the value of n that makes $\triangle DEF \sim \triangle XYZ$ when $DE = 4$, $EF = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle E \cong \angle Y$. Include a sketch.

15. **MAKING AN ARGUMENT** Your friend claims that $\triangle JKL \sim \triangle MNO$ by the SAS Similarity Theorem when $JK = 18$, $m\angle K = 130^\circ$, $KL = 16$, $MN = 9$, $m\angle N = 65^\circ$, and $NO = 8$. Do you support your friend's claim? Explain your reasoning.

16. **ANALYZING RELATIONSHIPS** Certain sections of stained glass are sold in triangular, beveled pieces. Which of the three beveled pieces, if any, are similar?



17. **ATTENDING TO PRECISION** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true? Select all that apply. Explain your reasoning.



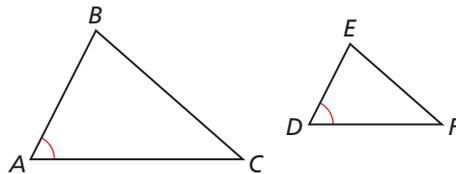
- (A) $\angle 1 \cong \angle 2$ (B) $\overline{QR} \parallel \overline{NP}$
 (C) $\angle 1 \cong \angle 4$ (D) $\triangle MNP \sim \triangle MRQ$

18. **WRITING** Are any two right triangles similar? Explain.

19. **PROVING A THEOREM** Write a two-column proof of the SAS Similarity Theorem.

Given $\angle A \cong \angle D$, $\frac{AB}{DE} = \frac{AC}{DF}$

Prove $\triangle ABC \sim \triangle DEF$



20. **HOW DO YOU SEE IT?** Which theorem could you use to show that $\triangle OPQ \sim \triangle OMN$ in the portion of the Ferris wheel shown when $PM = QN = 5$ feet and $MO = NO = 10$ feet?



21. **DRAWING CONCLUSIONS** Explain why it is not necessary to have an Angle-Side-Angle Similarity Theorem.

22. **THOUGHT PROVOKING** Decide whether each is a valid method of showing that two quadrilaterals are similar. Justify your answer.

- a. SASA b. SASAS c. SSSS d. SASSS

23. **WRITING** Can two triangles have all three ratios of corresponding angle measures equal to a value greater than 1? less than 1? Explain.

24. **MULTIPLE REPRESENTATIONS** Use a diagram to show why there is no Side-Side-Angle Similarity Theorem.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the slope of the line that passes through the given points. (*Skills Review Handbook*)

25. $(-3, 6), (2, 1)$

26. $(-3, -5), (9, -1)$

27. $(1, -2), (8, 12)$

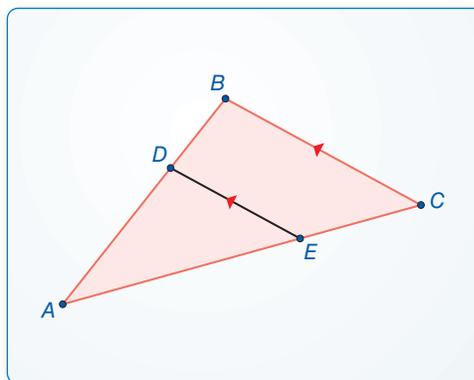
8.6 Proportionality Theorems

Essential Question What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

EXPLORATION 1 Discovering a Proportionality Relationship

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

- a. Construct \overline{DE} parallel to \overline{BC} with endpoints on \overline{AB} and \overline{AC} , respectively.



LOOKING FOR STRUCTURE

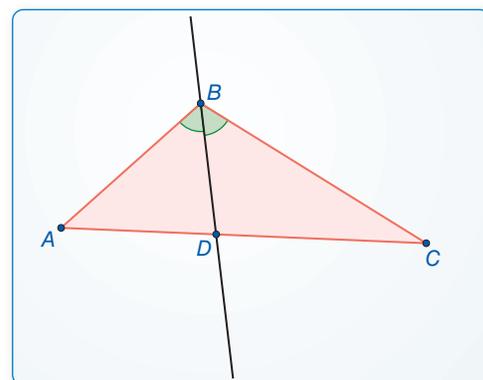
To be proficient in math, you need to look closely to discern a pattern or structure.

- b. Compare the ratios of AD to BD and AE to CE .
- c. Move \overline{DE} to other locations parallel to \overline{BC} with endpoints on \overline{AB} and \overline{AC} , and repeat part (b).
- d. Change $\triangle ABC$ and repeat parts (a)–(c) several times. Write a conjecture that summarizes your results.

EXPLORATION 2 Discovering a Proportionality Relationship

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

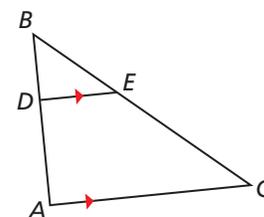
- a. Bisect $\angle B$ and plot point D at the intersection of the angle bisector and \overline{AC} .



- b. Compare the ratios of AD to DC and BA to BC .
- c. Change $\triangle ABC$ and repeat parts (a) and (b) several times. Write a conjecture that summarizes your results.

Communicate Your Answer

3. What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?
4. Use the figure at the right to write a proportion.



8.6 Lesson

Core Vocabulary

directed line segment, p. 502

Previous

corresponding angles
ratio
proportion

What You Will Learn

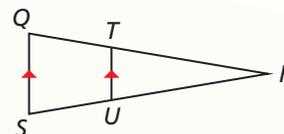
- ▶ Use proportionality theorems.
- ▶ Partition directed line segments.

Using Proportionality Theorems

Theorems

Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

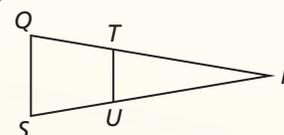


$$\text{If } \overline{TU} \parallel \overline{QS}, \text{ then } \frac{RT}{TQ} = \frac{RU}{US}.$$

Proof Ex. 27, p. 505

Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

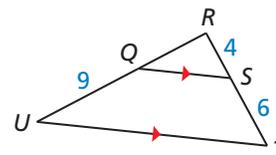


$$\text{If } \frac{RT}{TQ} = \frac{RU}{US}, \text{ then } \overline{TU} \parallel \overline{QS}.$$

Proof Ex. 28, p. 505

EXAMPLE 1 Finding the Length of a Segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of \overline{RQ} ?



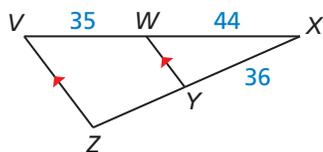
SOLUTION

$$\frac{RQ}{QU} = \frac{RS}{ST} \quad \text{Triangle Proportionality Theorem}$$

$$\frac{RQ}{9} = \frac{4}{6} \quad \text{Substitute.}$$

$$RQ = 6 \quad \text{Multiply each side by 9 and simplify.}$$

- ▶ The length of \overline{RQ} is 6 units.



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1. Find the length of \overline{YZ} .

The theorems above also imply the following:

Contrapositive of the Triangle Proportionality Theorem

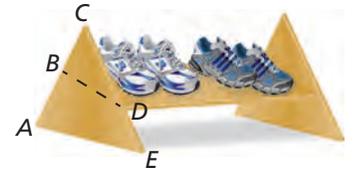
$$\text{If } \frac{RT}{TQ} \neq \frac{RU}{US}, \text{ then } \overline{TU} \not\parallel \overline{QS}.$$

Inverse of the Triangle Proportionality Theorem

$$\text{If } \overline{TU} \not\parallel \overline{QS}, \text{ then } \frac{RT}{TQ} \neq \frac{RU}{US}.$$

EXAMPLE 2 Solving a Real-Life Problem

On the shoe rack shown, $BA = 33$ centimeters, $CB = 27$ centimeters, $CD = 44$ centimeters, and $DE = 25$ centimeters. Explain why the shelf is not parallel to the floor.



SOLUTION

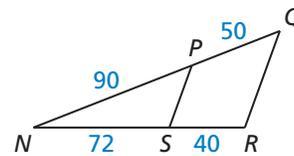
Find and simplify the ratios of the lengths.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

▶ Because $\frac{44}{25} \neq \frac{9}{11}$, \overline{BD} is not parallel to \overline{AE} . So, the shelf is not parallel to the floor.

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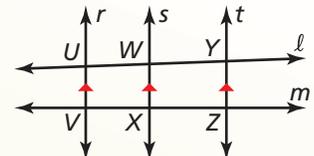
2. Determine whether $\overline{PS} \parallel \overline{QR}$.



Theorem

Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

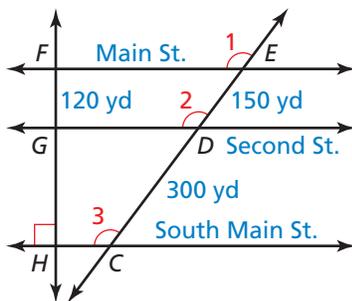


Proof Ex. 32, p. 505

$$\frac{UW}{WY} = \frac{VX}{XZ}$$

EXAMPLE 3 Using the Three Parallel Lines Theorem

In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent, $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



SOLUTION

Corresponding angles are congruent, so \overrightarrow{FE} , \overrightarrow{GD} , and \overrightarrow{HC} are parallel.

Use the Three Parallel Lines Theorem to set up a proportion.

$$\frac{HG}{GF} = \frac{CD}{DE} \qquad \text{Three Parallel Lines Theorem}$$

$$\frac{HG}{120} = \frac{300}{150} \qquad \text{Substitute.}$$

$$HG = 240 \qquad \text{Multiply each side by 120 and simplify.}$$

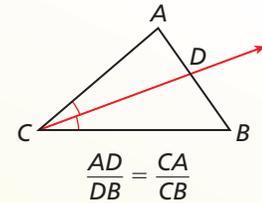
By the Segment Addition Postulate, $HF = HG + GF = 240 + 120 = 360$.

▶ The distance between Main Street and South Main Street is 360 yards.

Theorem

Triangle Angle Bisector Theorem

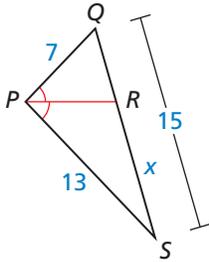
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



Proof Ex. 35, p. 506

EXAMPLE 4 Using the Triangle Angle Bisector Theorem

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of \overline{RS} .



SOLUTION

Because \overrightarrow{PR} is an angle bisector of $\angle QPS$, you can apply the Triangle Angle Bisector Theorem. Let $RS = x$. Then $RQ = 15 - x$.

$$\frac{RQ}{RS} = \frac{PQ}{PS}$$

Triangle Angle Bisector Theorem

$$\frac{15 - x}{x} = \frac{7}{13}$$

Substitute.

$$195 - 13x = 7x$$

Cross Products Property

$$9.75 = x$$

Solve for x .

► The length of \overline{RS} is 9.75 units.

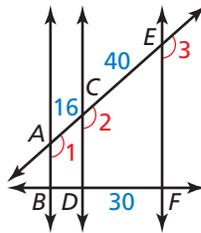
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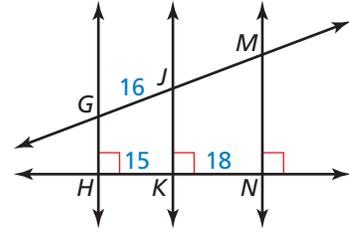
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Find the length of the given line segment.

3. \overline{BD}

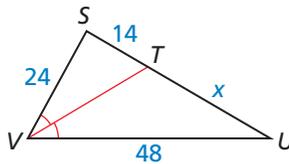


4. \overline{JM}

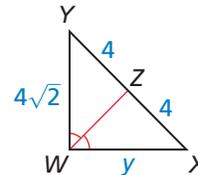


Find the value of the variable.

5.



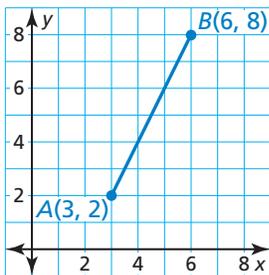
6.



Partitioning a Directed Line Segment

A **directed line segment** \overrightarrow{AB} is a segment that represents moving from point A to point B . The following example shows how to use slope to find a point on a directed line segment that partitions the segment in a given ratio.

EXAMPLE 5 Partitioning a Directed Line Segment



Find the coordinates of point P along the directed line segment \overline{AB} so that the ratio of AP to PB is 3 to 2.

SOLUTION

In order to divide the segment in the ratio 3 to 2, think of dividing, or *partitioning*, the segment into $3 + 2$, or 5 congruent pieces. Point P is the point that is $\frac{3}{5}$ of the way from point A to point B .

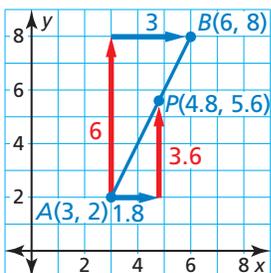
Find the rise and run from point A to point B . Leave the slope in terms of rise and run and do not simplify.

$$\text{slope of } \overline{AB}: m = \frac{8 - 2}{6 - 3} = \frac{6}{3} = \frac{\text{rise}}{\text{run}}$$

To find the coordinates of point P , add $\frac{3}{5}$ of the run to the x -coordinate of A , and add $\frac{3}{5}$ of the rise to the y -coordinate of A .

$$\text{run: } \frac{3}{5} \text{ of } 3 = \frac{3}{5} \cdot 3 = 1.8 \qquad \text{rise: } \frac{3}{5} \text{ of } 6 = \frac{3}{5} \cdot 6 = 3.6$$

► So, the coordinates of P are $(3 + 1.8, 2 + 3.6) = (4.8, 5.6)$. The ratio of AP to PB is 3 to 2.



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Find the coordinates of point P along the directed line segment \overline{AB} so that AP to PB is the given ratio.

7. $A(1, 3), B(8, 4)$; 4 to 1

8. $A(-2, 1), B(4, 5)$; 3 to 7

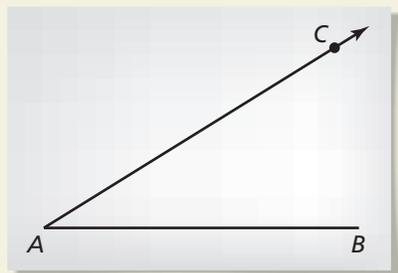
You can apply the Triangle Proportionality Theorem to construct a point along a directed line segment that partitions the segment in a given ratio.

CONSTRUCTION Constructing a Point along a Directed Line Segment

Construct the point L on \overline{AB} so that the ratio of AL to LB is 3 to 1.

SOLUTION

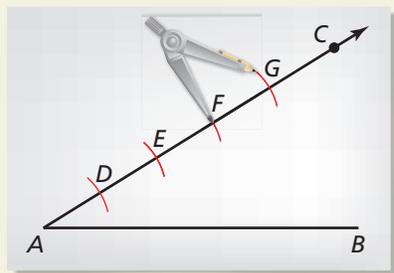
Step 1



Draw a segment and a ray

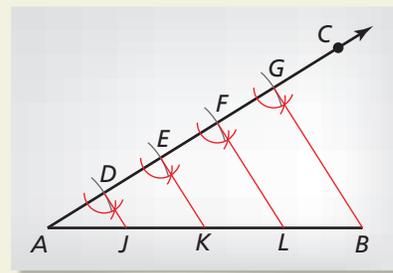
Draw \overline{AB} of any length. Choose any point C not on \overline{AB} . Draw \overrightarrow{AC} .

Step 2



Draw arcs Place the point of a compass at A and make an arc of any radius intersecting \overline{AC} . Label the point of intersection D . Using the same compass setting, make three more arcs on \overline{AC} , as shown. Label the points of intersection E, F , and G and note that $AD = DE = EF = FG$.

Step 3



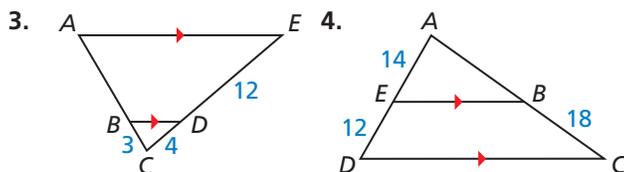
Draw a segment Draw \overline{GB} . Copy $\angle AGB$ and construct congruent angles at D, E , and F with sides that intersect \overline{AB} at J, K , and L . Sides $\overline{DJ}, \overline{EK}$, and \overline{FL} are all parallel, and they divide \overline{AB} equally. So, $AJ = JK = KL = LB$. Point L divides directed line segment \overline{AB} in the ratio 3 to 1.

Vocabulary and Core Concept Check

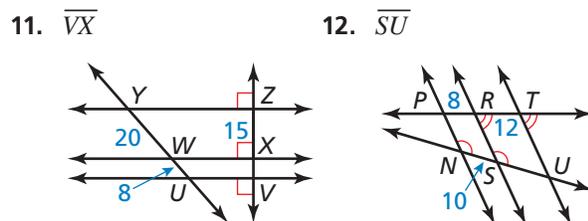
- COMPLETE THE STATEMENT** If a line divides two sides of a triangle proportionally, then it is _____ to the third side. This theorem is known as the _____.
- VOCABULARY** In $\triangle ABC$, point R lies on \overline{BC} and \overline{AR} bisects $\angle CAB$. Write the proportionality statement for the triangle that is based on the Triangle Angle Bisector Theorem.

Monitoring Progress and Modeling with Mathematics

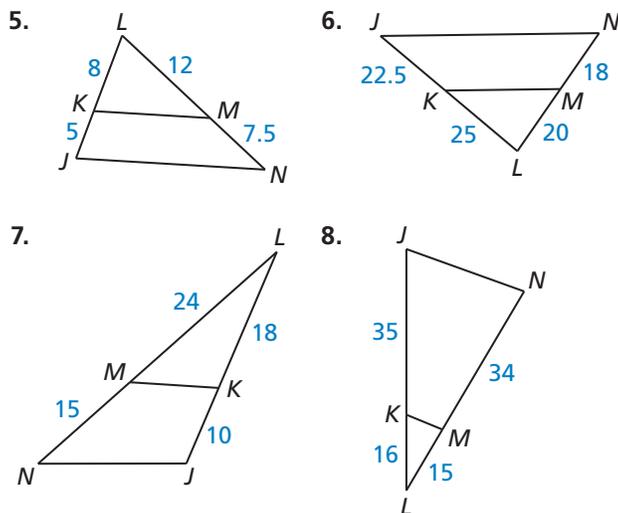
In Exercises 3 and 4, find the length of \overline{AB} .
(See Example 1.)



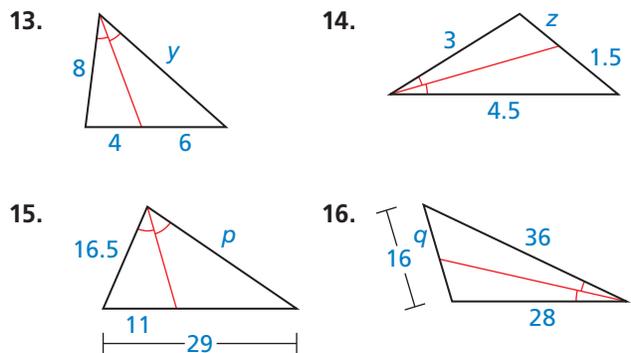
In Exercises 11 and 12, find the length of the indicated line segment. (See Example 3.)



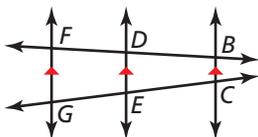
In Exercises 5–8, determine whether $\overline{KM} \parallel \overline{JN}$.
(See Example 2.)



In Exercises 13–16, find the value of the variable.
(See Example 4.)



In Exercises 9 and 10, use the diagram to complete the proportion.



9. $\frac{BD}{BF} = \frac{\square}{CG}$ 10. $\frac{CG}{\square} = \frac{BF}{DF}$

In Exercises 17–20, find the coordinates of point P along the directed line segment \overline{AB} so that AP to PB is the given ratio. (See Example 5.)

- $A(8, 0), B(3, -2); 1$ to 4
- $A(-2, -4), B(6, 1); 3$ to 2
- $A(1, 6), B(-2, -3); 5$ to 1
- $A(-3, 2), B(5, -4); 2$ to 6

CONSTRUCTION In Exercises 21 and 22, draw a segment with the given length. Construct the point that divides the segment in the given ratio.

- 3 in.; 1 to 4
- 2 in.; 2 to 3

23. **ERROR ANALYSIS** Describe and correct the error in solving for x .

X

$$\frac{AB}{BC} = \frac{CD}{AD} \Rightarrow \frac{10}{16} = \frac{14}{x}$$

$$10x = 224$$

$$x = 22.4$$

24. **ERROR ANALYSIS** Describe and correct the error in the student's reasoning.

X

Because $\frac{BD}{CD} = \frac{AB}{AC}$ and $BD = CD$,
 it follows that $AB = AC$.

MATHEMATICAL CONNECTIONS In Exercises 25 and 26, find the value of x for which $PQ \parallel RS$.

25.

26.

27. **PROVING A THEOREM** Prove the Triangle Proportionality Theorem.

Given $\overline{QS} \parallel \overline{TU}$

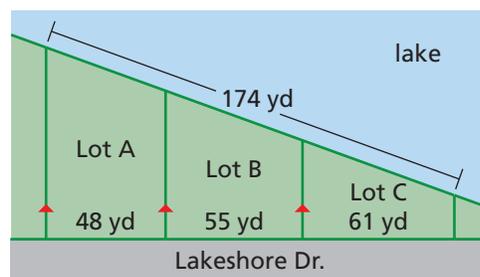
Prove $\frac{QT}{TR} = \frac{SU}{UR}$

28. **PROVING A THEOREM** Prove the Converse of the Triangle Proportionality Theorem.

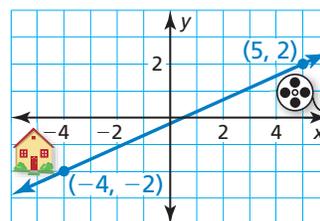
Given $\frac{ZY}{YW} = \frac{ZX}{XV}$

Prove $\overline{YX} \parallel \overline{WV}$

29. **MODELING WITH MATHEMATICS** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.



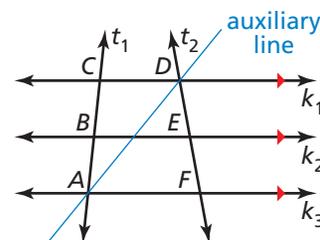
- Find the lake frontage (to the nearest tenth) of each lot shown.
 - In general, the more lake frontage a lot has, the higher its selling price. Which lot(s) should be listed for the highest price?
 - Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is \$250,000, what are the prices of the other lots? Explain your reasoning.
30. **MODELING WITH MATHEMATICS** Your school lies directly between your house and the movie theater. The distance from your house to the school is one-fourth of the distance from the school to the movie theater. What point on the graph represents your school?



31. **REASONING** In the construction on page 503, explain why you can apply the Triangle Proportionality Theorem in Step 3.
32. **PROVING A THEOREM** Use the diagram with the auxiliary line drawn to write a paragraph proof of the Three Parallel Lines Theorem.

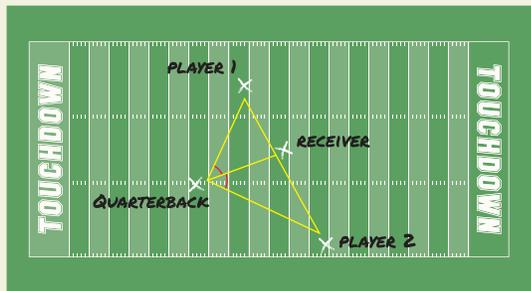
Given $k_1 \parallel k_2 \parallel k_3$

Prove $\frac{CB}{BA} = \frac{DE}{EF}$



33. CRITICAL THINKING In $\triangle LMN$, the angle bisector of $\angle M$ also bisects \overline{LN} . Classify $\triangle LMN$ as specifically as possible. Justify your answer.

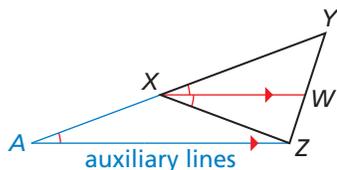
34. HOW DO YOU SEE IT? During a football game, the quarterback throws the ball to the receiver. The receiver is between two defensive players, as shown. If Player 1 is closer to the quarterback when the ball is thrown and both defensive players move at the same speed, which player will reach the receiver first? Explain your reasoning.



35. PROVING A THEOREM Use the diagram with the auxiliary lines drawn to write a paragraph proof of the Triangle Angle Bisector Theorem.

Given $\angle YXW \cong \angle WXZ$

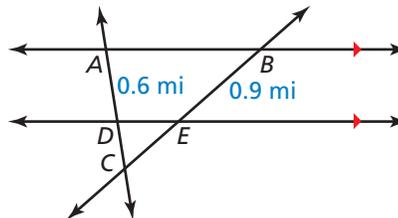
Prove $\frac{YW}{WZ} = \frac{XY}{XZ}$



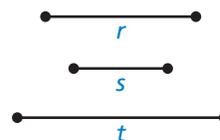
36. THOUGHT PROVOKING Write the converse of the Triangle Angle Bisector Theorem. Is the converse true? Justify your answer.

37. REASONING How is the Triangle Midsegment Theorem related to the Triangle Proportionality Theorem? Explain your reasoning.

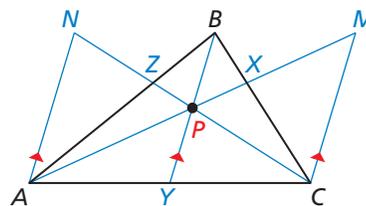
38. MAKING AN ARGUMENT Two people leave points A and B at the same time. They intend to meet at point C at the same time. The person who leaves point A walks at a speed of 3 miles per hour. You and a friend are trying to determine how fast the person who leaves point B must walk. Your friend claims you need to know the length of \overline{AC} . Is your friend correct? Explain your reasoning.



39. CONSTRUCTION Given segments with lengths r , s , and t , construct a segment of length x , such that $\frac{r}{s} = \frac{t}{x}$.



40. PROOF Prove Ceva's Theorem: If P is any point inside $\triangle ABC$, then $\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$.



(Hint: Draw segments parallel to \overline{BY} through A and C , as shown. Apply the Triangle Proportionality Theorem to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)

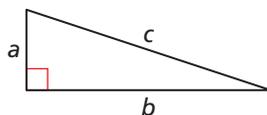
Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the triangle. (Skills Review Handbook)

41. Which sides are the legs?

42. Which side is the hypotenuse?



Solve the equation. (Section 4.3)

43. $x^2 = 121$

44. $x^2 + 16 = 25$

45. $36 + x^2 = 85$

8.5–8.6 What Did You Learn?

Core Vocabulary

directed line segment, *p.* 502

Core Concepts

Section 8.5

Side-Side-Side (SSS) Similarity Theorem, *p.* 494

Side-Angle-Side (SAS) Similarity Theorem, *p.* 496

Section 8.6

Triangle Proportionality Theorem, *p.* 500

Converse of the Triangle Proportionality Theorem, *p.* 500

Three Parallel Lines Theorem, *p.* 501

Triangle Angle Bisector Theorem, *p.* 502

Partitioning a Directed Line Segment, *p.* 503

Mathematical Practices

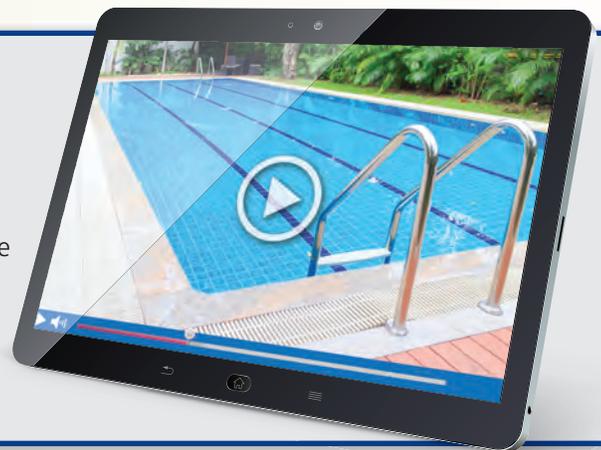
1. In Exercise 16 on page 498, why do you only need to find two ratios instead of three to compare the side lengths of a pair of triangles?
2. In Exercise 4 on page 504, is it better to use $\frac{7}{6}$ or 1.17 as your ratio of the lengths when finding the length of \overline{AB} ? Explain your reasoning.
3. In Exercise 30 on page 505, a classmate tells you that your answer is incorrect because you should have divided the segment into four congruent pieces. Respond to your classmate's argument by justifying your original answer.

Performance Task:

Pool Maintenance

Your pool service company has been very successful at maintaining rectangular pools designed for regulation competitions. Now you want to expand your services to owners of private, recreational pools. How will your experience transfer to pools of various shapes and sizes?

To explore the answer to this question and more, check out the Performance Task and Real-Life STEM video at BigIdeasMath.com.



8.1 Dilations (pp. 461–468)

Graph trapezoid $ABCD$ with vertices $A(1, 1)$, $B(1, 3)$, $C(3, 2)$, and $D(3, 1)$ and its image after a dilation with a scale factor of 2.

Use the coordinate rule for a dilation with $k = 2$ to find the coordinates of the vertices of the image. Then graph trapezoid $ABCD$ and its image.

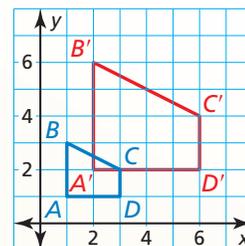
$$(x, y) \rightarrow (2x, 2y)$$

$$A(1, 1) \rightarrow A'(2, 2)$$

$$B(1, 3) \rightarrow B'(2, 6)$$

$$C(3, 2) \rightarrow C'(6, 4)$$

$$D(3, 1) \rightarrow D'(6, 2)$$



Graph the triangle and its image after a dilation with scale factor k .

- $P(2, 2)$, $Q(4, 4)$, $R(8, 2)$; $k = \frac{1}{2}$
- $X(-3, 2)$, $Y(2, 3)$, $Z(1, -1)$; $k = -3$
- You are using a magnifying glass that shows the image of an object that is eight times the object's actual size. The image length is 15.2 centimeters. Find the actual length of the object.

8.2 Similarity and Transformations (pp. 469–474)

Describe a similarity transformation that maps $\triangle FGH$ to $\triangle LMN$, as shown at the right.

\overline{FG} is horizontal, and \overline{LM} is vertical. If you rotate $\triangle FGH$ 90° about the origin as shown at the bottom right, then the image, $\triangle F'G'H'$, will have the same orientation as $\triangle LMN$. $\triangle LMN$ appears to be half as large as $\triangle F'G'H'$. Dilate $\triangle F'G'H'$ using a scale factor of $\frac{1}{2}$.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

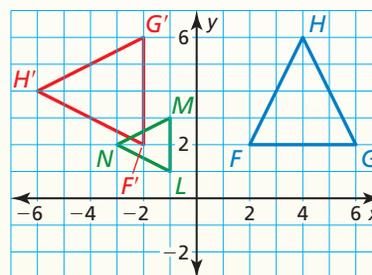
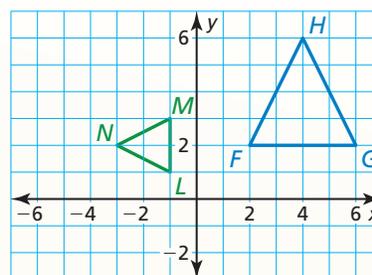
$$F'(-2, 2) \rightarrow F''(-1, 1)$$

$$G'(-2, 6) \rightarrow G''(-1, 3)$$

$$H'(-6, 4) \rightarrow H''(-3, 2)$$

The vertices of $\triangle F''G''H''$ match the vertices of $\triangle LMN$.

- So, a similarity transformation that maps $\triangle FGH$ to $\triangle LMN$ is a rotation of 90° about the origin followed by a dilation with a scale factor of $\frac{1}{2}$.



Describe a similarity transformation that maps $\triangle ABC$ to $\triangle RST$.

- $A(1, 0)$, $B(-2, -1)$, $C(-1, -2)$ and $R(-3, 0)$, $S(6, -3)$, $T(3, -6)$
- $A(6, 4)$, $B(-2, 0)$, $C(-4, 2)$ and $R(2, 3)$, $S(0, -1)$, $T(1, -2)$
- $A(3, -2)$, $B(0, 4)$, $C(-1, -3)$ and $R(-4, -6)$, $S(8, 0)$, $T(-6, 2)$

8.3 Similar Polygons (pp. 475–484)

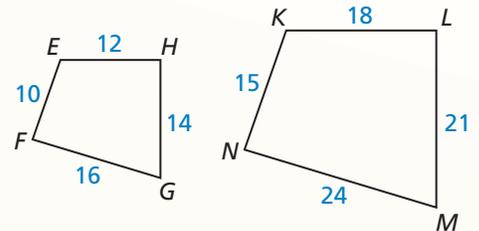
In the diagram, $EHGF \sim KLMN$. Find the scale factor from $EHGF$ to $KLMN$. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

From the diagram, you can see that \overline{EH} and \overline{KL} are corresponding sides. So, the scale factor of

$$EHGF \text{ to } KLMN \text{ is } \frac{KL}{EH} = \frac{18}{12} = \frac{3}{2}.$$

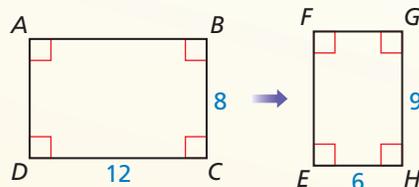
$\angle E \cong \angle K$, $\angle H \cong \angle L$, $\angle G \cong \angle M$, and $\angle F \cong \angle N$.

$$\frac{KL}{EH} = \frac{LM}{HG} = \frac{MN}{GF} = \frac{NK}{FE}$$

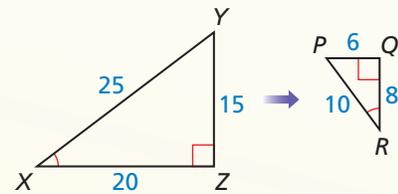


Find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

7. $ABCD \sim EFGH$



8. $\triangle XYZ \sim \triangle RPQ$



- Two similar triangles have a scale factor of 3 : 5. The altitude of the larger triangle is 24 inches. What is the altitude of the smaller triangle?
- Two similar triangles have a pair of corresponding sides of length 12 meters and 8 meters. The larger triangle has a perimeter of 48 meters and an area of 180 square meters. Find the perimeter and area of the smaller triangle.

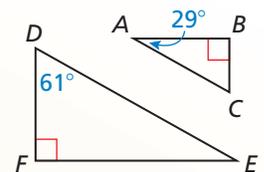
8.4 Proving Triangle Similarity by AA (pp. 485–490)

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

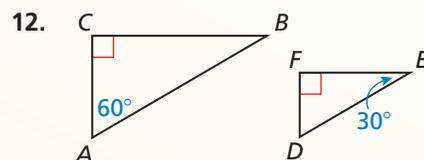
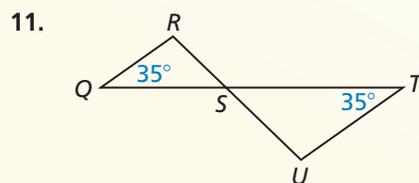
Because they are both right angles, $\angle F$ and $\angle B$ are congruent.

By the Triangle Sum Theorem, $61^\circ + 90^\circ + m\angle E = 180^\circ$, so $m\angle E = 29^\circ$.

So, $\angle E$ and $\angle A$ are congruent. So, $\triangle DFE \sim \triangle CBA$ by the AA Similarity Theorem.



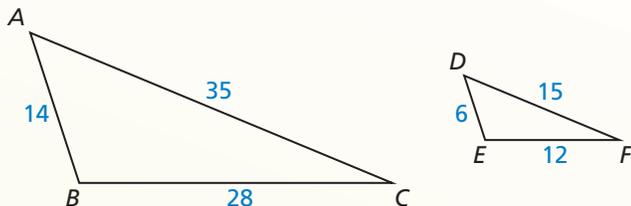
Show that the triangles are similar. Write a similarity statement.



- A cellular telephone tower casts a shadow that is 72 feet long, while a nearby tree that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

8.5 Proving Triangle Similarity by SSS and SAS (pp. 493–498)

Show that the triangles are similar.



Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides

$$\frac{AB}{DE} = \frac{14}{6} = \frac{7}{3}$$

Longest sides

$$\frac{AC}{DF} = \frac{35}{15} = \frac{7}{3}$$

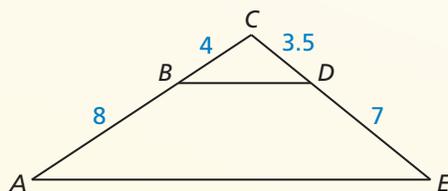
Remaining sides

$$\frac{BC}{EF} = \frac{28}{12} = \frac{7}{3}$$

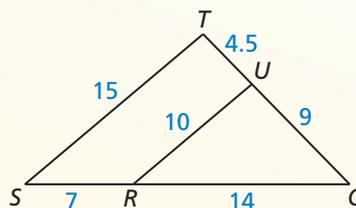
▶ All the ratios are equal, so $\triangle ABC \sim \triangle DEF$ by the SSS Similarity Theorem.

Use the SSS Similarity Theorem or the SAS Similarity Theorem to show that the triangles are similar.

14.



15.



8.6 Proportionality Theorems (pp. 499–506)

In the diagram, \overline{AD} bisects $\angle CAB$. Find the length of \overline{DB} .

Because \overline{AD} is an angle bisector of $\angle CAB$, you can apply the Triangle Angle Bisector Theorem.

$$\frac{DB}{DC} = \frac{AB}{AC}$$

Triangle Angle Bisector Theorem

$$\frac{x}{5} = \frac{15}{8}$$

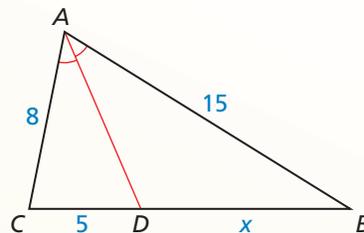
Substitute.

$$8x = 75$$

Cross Products Property

$$9.375 = x$$

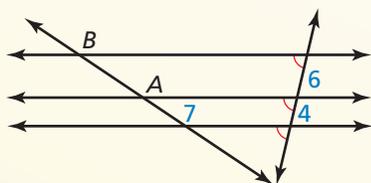
Solve for x .



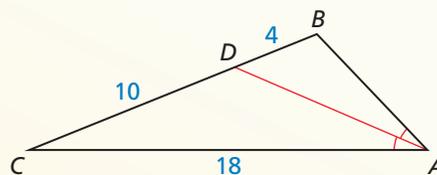
▶ The length of \overline{DB} is 9.375 units.

Find the length of \overline{AB} .

16.

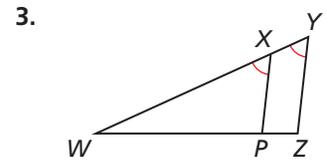
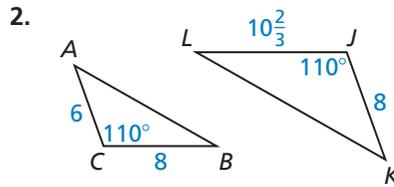
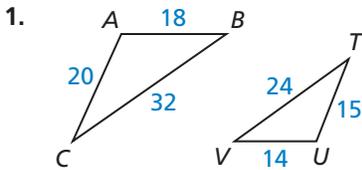


17.

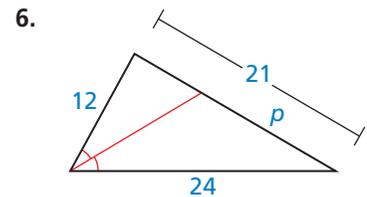
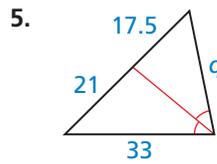
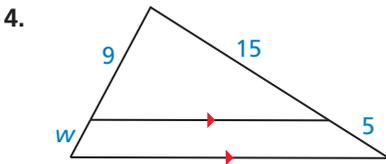


8 Chapter Test

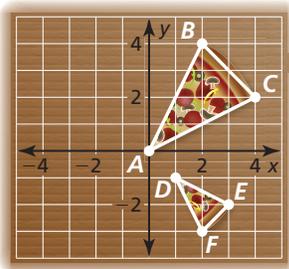
Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Find the value of the variable.



7. Given $\triangle QRS \sim \triangle MNP$, list all pairs of congruent angles. Then write the ratios of the corresponding side lengths in a statement of proportionality.

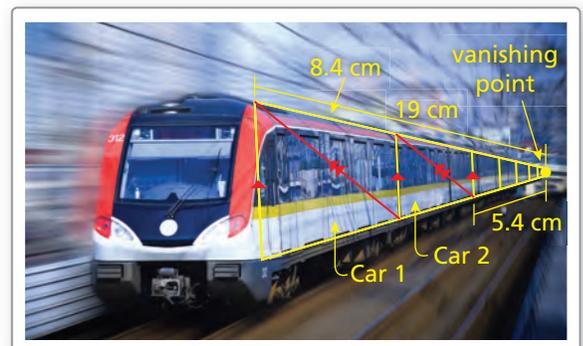


8. There is one slice of a large pizza and one slice of a small pizza in the box.
- Describe a similarity transformation that maps pizza slice ABC to pizza slice DEF .
 - What is one possible scale factor for a medium slice of pizza? Explain your reasoning. (Use a dilation on the large slice of pizza.)

9. The original photograph shown is 4 inches by 6 inches.
- What transformations can you use to produce the new photograph?
 - You dilate the original photograph by a scale factor of $\frac{1}{2}$. What are the dimensions of the new photograph?
 - You have a frame that holds photos that are 8.5 inches by 11 inches. Can you dilate the original photograph to fit the frame? Explain your reasoning.



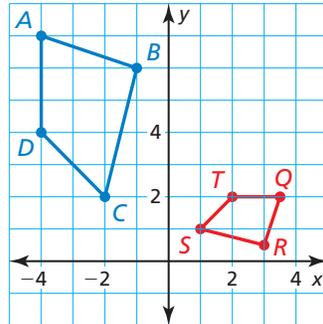
10. In a *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel. Find the length of the bottom edge of the drawing of Car 2.



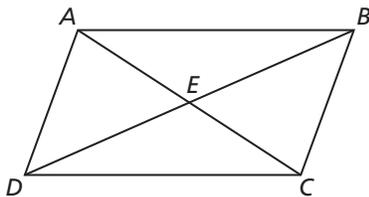
8

Cumulative Assessment

1. Use the graph of quadrilaterals $ABCD$ and $QRST$.



- Write a composition of transformations that maps quadrilateral $ABCD$ to quadrilateral $QRST$.
 - Are the quadrilaterals similar? Explain your reasoning.
2. In the diagram, $ABCD$ is a parallelogram. Which congruence theorem(s) could you use to show that $\triangle AED \cong \triangle CEB$? Select all that apply.



SAS Congruence Theorem

SSS Congruence Theorem

HL Congruence Theorem

ASA Congruence Theorem

AAS Congruence Theorem

3. Which expression is *not* equivalent to $20x^2 + 27x - 14$?

- (A) $27x + 20x^2 - 14$
 (B) $(5x - 2)(4x + 7)$
 (C) $(5x + 2)(4x - 7)$
 (D) $(4x + 7)(5x - 2)$

4. A bag contains six tiles with a letter on each tile. The letters are A, B, C, D, E, and F. You randomly draw two tiles from the bag. Which expression represents the probability that you draw the A tile and the B tile?

$\frac{2!}{6!}$

$\frac{1}{6P_2}$

$6P_2$

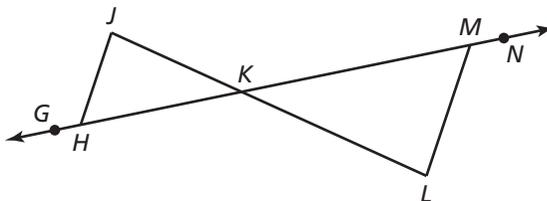
$\frac{1}{6C_2}$

$6C_2$

$\frac{6C_2}{6C_4}$

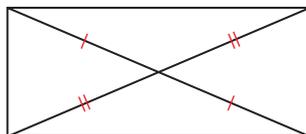
5. Enter a statement or reason in each blank to complete the two-column proof.

Given $\frac{KJ}{KL} = \frac{KH}{KM}$
Prove $\angle LMN \cong \angle JHG$



STATEMENTS	REASONS
1. $\frac{KJ}{KL} = \frac{KH}{KM}$	1. Given
2. $\angle JKH \cong \angle LKM$	2. _____
3. $\triangle JKH \sim \triangle LKM$	3. _____
4. $\angle KHJ \cong \angle KML$	4. _____
5. _____	5. Definition of congruent angles
6. $m\angle KHJ + m\angle JHG = 180^\circ$	6. Linear Pair Postulate
7. $m\angle JHG = 180^\circ - m\angle KHJ$	7. _____
8. $m\angle KML + m\angle LMN = 180^\circ$	8. _____
9. _____	9. Subtraction Property of Equality
10. $m\angle LMN = 180^\circ - m\angle KML$	10. _____
11. _____	11. Transitive Property of Equality
12. $\angle LMN \cong \angle JHG$	12. _____

6. The coordinates of the vertices of $\triangle DEF$ are $D(-8, 5)$, $E(-5, 8)$, and $F(-1, 4)$. The coordinates of the vertices of $\triangle JKL$ are $J(16, -10)$, $K(10, -16)$, and $L(2, -8)$. $\angle D \cong \angle J$. Can you show that $\triangle DEF \sim \triangle JKL$ by using the AA Similarity Theorem? If so, do so by listing the congruent corresponding angles and writing a similarity transformation that maps $\triangle DEF$ to $\triangle JKL$. If not, explain why not.
7. Classify the quadrilateral using the most specific name.



rectangle

square

parallelogram

rhombus

8. Your friend makes the statement “Quadrilateral $PQRS$ is similar to quadrilateral $WXYZ$.” Describe the relationships between corresponding angles and between corresponding sides that make this statement true.