2 Polynomial Equations and Factoring

Height of a Falling Object *(p. 104)*

Game Reserve *(p. 98)*

Photo Cropping *(p. 94)*

Gateway Arch *(p. 86)*

Framing a Photo *(p. 74)*

Maintaining Mathematical Proficiency

Simplifying Algebraic Expressions

Example 1 **Simplify $6x + 5 - 3x - 4$.**

$$\begin{aligned} 6x + 5 - 3x - 4 &= 6x - 3x + 5 - 4 && \text{Commutative Property of Addition} \\ &= (6 - 3)x + 5 - 4 && \text{Distributive Property} \\ &= 3x + 1 && \text{Simplify.} \end{aligned}$$

Example 2 **Simplify $-8(y - 3) + 2y$.**

$$\begin{aligned} -8(y - 3) + 2y &= -8(y) - (-8)(3) + 2y && \text{Distributive Property} \\ &= -8y + 24 + 2y && \text{Multiply.} \\ &= -8y + 2y + 24 && \text{Commutative Property of Addition} \\ &= (-8 + 2)y + 24 && \text{Distributive Property} \\ &= -6y + 24 && \text{Simplify.} \end{aligned}$$

Simplify the expression.

1. $3x - 7 + 2x$ **2.** $4r + 6 - 9r - 1$ **3.** $-5t + 3 - t - 4 + 8t$

4. $3(s - 1) + 5$ **5.** $2m - 7(3 - m)$ **6.** $4(h + 6) - (h - 2)$

Finding the Greatest Common Factor

Example 3 **Find the greatest common factor (GCF) of 42 and 70.**

To find the GCF of two numbers, first write the prime factorization of each number. Then find the product of the common prime factors.

$$42 = 2 \cdot 3 \cdot 7$$

$$70 = 2 \cdot 5 \cdot 7$$

▶ The GCF of 42 and 70 is $2 \cdot 7 = 14$.

Find the greatest common factor.

7. 20, 36 **8.** 42, 63 **9.** 54, 81

10. 72, 84 **11.** 28, 64 **12.** 30, 77

13. **ABSTRACT REASONING** Is it possible for two integers to have no common factors? Explain your reasoning.

Mathematical Practices

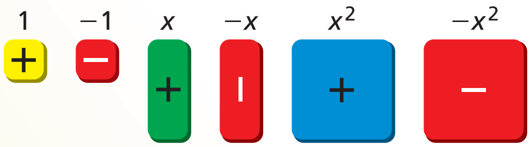
Mathematically proficient students consider concrete models when solving a mathematics problem.

Using Models

Core Concept

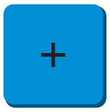
Using Algebra Tiles

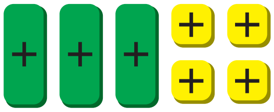
When solving a problem, it can be helpful to use a model. For instance, you can use algebra tiles to model algebraic expressions and operations with algebraic expressions.

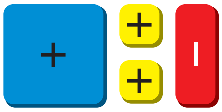


EXAMPLE 1 Writing Expressions Modeled by Algebra Tiles

Write the algebraic expression modeled by the algebra tiles.

a. 

b. 

c. 

SOLUTION

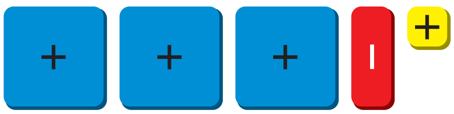
a. The algebraic expression is x^2.

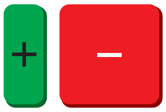
b. The algebraic expression is $3x + 4$.

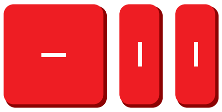
c. The algebraic expression is $x^2 - x + 2$.

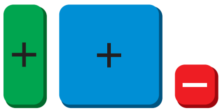
Monitoring Progress

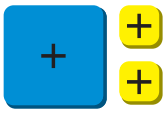
Write the algebraic expression modeled by the algebra tiles.

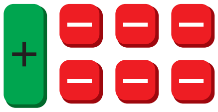
1. 

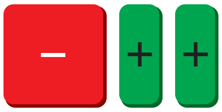
2. 

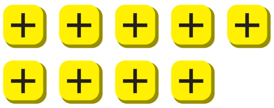
3. 

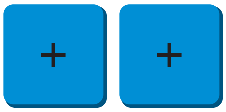
4. 

5. 

6. 

7. 

8. 

9. 

2.1 Adding and Subtracting Polynomials

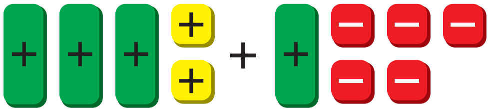
Essential Question

How can you add and subtract polynomials?

EXPLORATION 1 Adding Polynomials

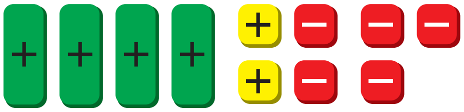
Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1

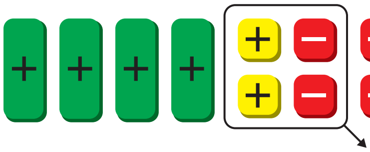


$(3x + 2) + (x - 5)$

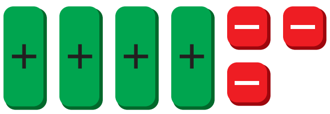
Step 2



Step 3



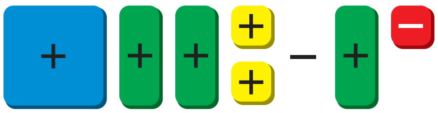
Step 4



EXPLORATION 2 Subtracting Polynomials

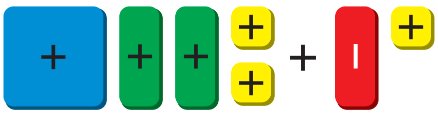
Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1

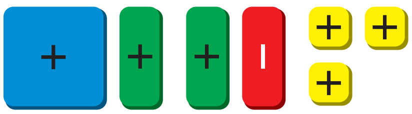


$(x^2 + 2x + 2) - (x - 1)$

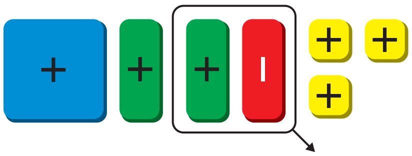
Step 2



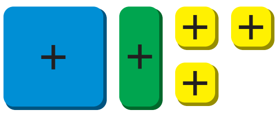
Step 3



Step 4



Step 5



REASONING ABSTRACTLY

To be proficient in math, you need to represent a given situation using symbols.

Communicate Your Answer

3. How can you add and subtract polynomials?

4. Use your methods in Question 3 to find each sum or difference.

a. $(x^2 + 2x - 1) + (2x^2 - 2x + 1)$ **b.** $(4x + 3) + (x - 2)$

c. $(x^2 + 2) - (3x^2 + 2x + 5)$ **d.** $(2x - 3x) - (x^2 - 2x + 4)$

2.1 Lesson

Core Vocabulary

monomial, *p. 62*
degree of a monomial, *p. 62*
polynomial, *p. 63*
binomial, *p. 63*
trinomial, *p. 63*
degree of a polynomial, *p. 63*
standard form, *p. 63*
leading coefficient, *p. 63*
closed, *p. 64*

What You Will Learn

- Find the degrees of monomials.
- Classify polynomials.
- Add and subtract polynomials.
- Solve real-life problems.

Finding the Degrees of Monomials

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

Monomial	Degree
10	0
$3x$	1
$\frac{1}{2}ab^2$	$1 + 2 = 3$
$-1.8m^5$	5

Not a monomial	Reason
$5 + x$	A sum is not a monomial.
$\frac{2}{n}$	A monomial cannot have a variable in the denominator.
4^a	A monomial cannot have a variable exponent.
x^{-1}	The variable must have a whole number exponent.

EXAMPLE 1 Finding the Degrees of Monomials

Find the degree of each monomial.

a. $5x^2$ **b.** $-\frac{1}{2}xy^3$ **c.** $8x^3y^3$ **d.** -3

SOLUTION

a. The exponent of x is 2.

▶ So, the degree of the monomial is 2.

b. The exponent of x is 1, and the exponent of y is 3.

▶ So, the degree of the monomial is $1 + 3$, or 4.

c. The exponent of x is 3, and the exponent of y is 3.

▶ So, the degree of the monomial is $3 + 3$, or 6.

d. You can rewrite -3 as $-3x^0$.

▶ So, the degree of the monomial is 0.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the degree of the monomial.

1. $-3x^4$ **2.** $7c^3d^2$ **3.** $\frac{5}{3}y$ **4.** -20.5

Classifying Polynomials

Core Concept

Polynomials

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial. A polynomial with two terms is a **binomial**. A polynomial with three terms is a **trinomial**.

Binomial	Trinomial
$5x + 2$	$x^2 + 5x + 2$

The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the **leading coefficient**.

leading coefficient   degree   constant term

$$2x^3 + x^2 - 5x + 12$$

EXAMPLE 2 Writing a Polynomial in Standard Form

Write $15x - x^3 + 3$ in standard form. Identify the degree and leading coefficient of the polynomial.

SOLUTION

Consider the degree of each term of the polynomial.

Degree is 1.   Degree is 3.   Degree is 0.

$$15x - x^3 + 3$$

▶ You can write the polynomial in standard form as $-x^3 + 15x + 3$. The greatest degree is 3, so the degree of the polynomial is 3, and the leading coefficient is -1.

EXAMPLE 3 Classifying Polynomials

Write each polynomial in standard form. Identify the degree and classify each polynomial by the number of terms.

a. $-3z^4$ **b.** $4 + 5x^2 - x$ **c.** $8q + q^5$

SOLUTION

Polynomial	Standard Form	Degree	Type of Polynomial
a. $-3z^4$	$-3z^4$	4	monomial
b. $4 + 5x^2 - x$	$5x^2 - x + 4$	2	trinomial
c. $8q + q^5$	$q^5 + 8q$	5	binomial

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

5. $4 - 9z$ **6.** $t^2 - t^3 - 10t$ **7.** $2.8x + x^3$

Adding and Subtracting Polynomials

A set of numbers is **closed** under an operation when the operation performed on any two numbers in the set results in a number that is also in the set. For example, the set of integers is closed under addition, subtraction, and multiplication. This means that if a and b are two integers, then $a + b$, $a - b$, and ab are also integers.

The set of polynomials is closed under addition and subtraction. So, the sum or difference of any two polynomials is also a polynomial.

To add polynomials, add like terms. You can use a vertical or a horizontal format.

EXAMPLE 4 Adding Polynomials

STUDY TIP

When a power of the variable appears in one polynomial but not the other, leave a space in that column, or write the term with a coefficient of 0.

Find the sum.

a. $(2x^3 - 5x^2 + x) + (2x^2 + x^3 - 1)$ **b.** $(3x^2 + x - 6) + (x^2 + 4x + 10)$

SOLUTION

a. Vertical format: Align like terms vertically and add.

$$\begin{array}{rrrrr} & 2x^3 & -\,5x^2 & +\,x & \\ + & x^3 & +\,2x^2 & & -\,1 \\ \hline & 3x^3 & -\,3x^2 & +\,x & -\,1 \end{array}$$

▶ The sum is $3x^3 - 3x^2 + x - 1$.

b. Horizontal format: Group like terms and simplify.

$$\begin{aligned}(3x^2 + x - 6) + (x^2 + 4x + 10) &= (3x^2 + x^2) + (x + 4x) + (-6 + 10)\\ &= 4x^2 + 5x + 4\end{aligned}$$

▶ The sum is $4x^2 + 5x + 4$.

To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by -1.

EXAMPLE 5 Subtracting Polynomials

COMMON ERROR

Remember to multiply *each* term of the polynomial by -1 when you write the subtraction as addition.

Find the difference.

a. $(4n^2 + 5) - (-2n^2 + 2n - 4)$ **b.** $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

SOLUTION

a. Vertical format: Align like terms vertically and subtract.

$$\begin{array}{rl} & 4n^2 \qquad\quad + 5 \\ - & (-2n^2 + 2n - 4) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rrrr} & 4n^2 & & +\,5 \\ + & 2n^2 & -\,2n & +\,4 \\ \hline & 6n^2 & -\,2n & +\,9 \end{array}$$

▶ The difference is $6n^2 - 2n + 9$.

b. Horizontal format: Group like terms and simplify.

$$\begin{aligned}(4x^2 - 3x + 5) - (3x^2 - x - 8) &= 4x^2 - 3x + 5 - 3x^2 + x + 8\\ &= (4x^2 - 3x^2) + (-3x + x) + (5 + 8)\\ &= x^2 - 2x + 13\end{aligned}$$

▶ The difference is $x^2 - 2x + 13$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the sum or difference.

8. $(b - 10) + (4b - 3)$

9. $(x^2 - x - 2) + (7x^2 - x)$

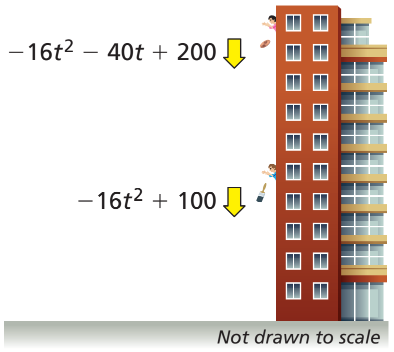
10. $(p^2 + p + 3) - (-4p^2 - p + 3)$

11. $(-k + 5) - (3k^2 - 6)$

Solving Real-Life Problems

EXAMPLE 6 Solving a Real-Life Problem

A penny is thrown straight down from a height of 200 feet. At the same time, a paintbrush is dropped from a height of 100 feet. The polynomials represent the heights (in feet) of the objects after t seconds.



a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.

b. Interpret the coefficients of the polynomial in part (a).

SOLUTION

a. To find the distance between the objects after t seconds, subtract the polynomials.

$$\begin{array}{lrcr} \textbf{Penny} & -16t^2 - 40t + 200 & & -16t^2 - 40t + 200 \\ \textbf{Paintbrush} & \underline{-\ (-16t^2 \qquad\quad + 100)} & \rightarrow & \underline{+ \quad 16t^2 \qquad\quad - 100} \\ & & & -40t + 100 \end{array}$$

▶ The polynomial $-40t + 100$ represents the distance between the objects after t seconds.

b. When $t = 0$, the distance between the objects is $-40(0) + 100 = 100$ feet. So, the constant term 100 represents the distance between the penny and the paintbrush when both objects begin to fall.

As the value of t increases by 1, the value of $-40t + 100$ decreases by 40. This means that the objects become 40 feet closer to each other each second. So, -40 represents the amount that the distance between the objects changes each second.

INTERPRETING EXPRESSIONS

Notice that each term of the resulting expression has special meaning in the context of the problem. Analyzing the terms helps you understand the problem in more depth.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

12. WHAT IF? The polynomial $-16t^2 - 25t + 200$ represents the height of the penny after t seconds.

a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.

b. Interpret the coefficients of the polynomial in part (a).

2.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** When is a polynomial in one variable in standard form?

2. **OPEN-ENDED** Write a trinomial in one variable of degree 5 in standard form.

3. **VOCABULARY** How can you determine whether a set of numbers is closed under an operation?

4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$a^3 + 4a$	$x^2 - 8^x$	$b - 2^{-1}$	$-\frac{\pi}{3} + 6y^8z$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, find the degree of the monomial. *(See Example 1.)*

5. $4g$
6. $23x^4$
7. $-1.75k^2$
8. $-\frac{4}{9}$
9. s^8t
10. $8m^2n^4$
11. $9xy^3z^7$
12. $-3q^4rs^6$

In Exercises 13–20, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms. *(See Examples 2 and 3.)*

13. $6c^2 + 2c^4 - c$
14. $4w^{11} - w^{12}$
15. $7 + 3p^2$
16. $8d - 2 - 4d^3$
17. $3t^8$
18. $5z + 2z^3 + 3z^4$
19. $\pi r^2 - \frac{5}{7}r^8 + 2r^5$
20. $\sqrt{7}n^4$

21. **MODELING WITH MATHEMATICS** The expression $\frac{4}{3}\pi r^3$ represents the volume of a sphere with radius r. Why is this expression a monomial? What is its degree?

22. **MODELING WITH MATHEMATICS** The amount of money you have after investing \$400 for 8 years and \$600 for 6 years at the same interest rate is represented by $400x^8 + 600x^6$, where x is the growth factor. Classify the polynomial by the number of terms. What is its degree?

In Exercises 23–30, find the sum. *(See Example 4.)*

23. $(5y + 4) + (-2y + 6)$
24. $(-8x - 12) + (9x + 4)$
25. $(2n^2 - 5n - 6) + (-n^2 - 3n + 11)$
26. $(-3p^3 + 5p^2 - 2p) + (-p^3 - 8p^2 - 15p)$
27. $(3g^2 - g) + (3g^2 - 8g + 4)$
28. $(9r^2 + 4r - 7) + (3r^2 - 3r)$
29. $(4a - a^3 - 3) + (2a^3 - 5a^2 + 8)$
30. $(s^3 - 2s - 9) + (2s^2 - 6s^3 + s)$

In Exercises 31–38, find the difference. *(See Example 5.)*

31. $(d - 9) - (3d - 1)$
32. $(6x + 9) - (7x + 1)$
33. $(y^2 - 4y + 9) - (3y^2 - 6y - 9)$
34. $(4m^2 - m + 2) - (-3m^2 + 10m + 4)$
35. $(k^3 - 7k + 2) - (k^2 - 12)$
36. $(-r - 10) - (-4r^3 + r^2 + 7r)$

37. $(t^4 - t^2 + t) - (12 - 9t^2 - 7t)$

38. $(4d - 6d^3 + 3d^2) - (10d^3 + 7d - 2)$

ERROR ANALYSIS In Exercises 39 and 40, describe and correct the error in finding the sum or difference.

39.

$$\begin{aligned}(x^2 + x) - (2x^2 - 3x) &= x^2 + x - 2x^2 - 3x \\ &= (x^2 - 2x^2) + (x - 3x) \\ &= -x^2 - 2x\end{aligned}$$

40.

$$\begin{array}{r} x^3 - 4x^2 + 3 \\ + \; -3x^3 + 8x \;\; - 2 \\ \hline -2x^3 + 4x^2 + 1 \end{array}$$

41. **MODELING WITH MATHEMATICS** The cost (in dollars) of making b bracelets is represented by $4 + 5b$. The cost (in dollars) of making b necklaces is represented by $8b + 6$. Write a polynomial that represents how much more it costs to make b necklaces than b bracelets.

42. **MODELING WITH MATHEMATICS** The number of individual memberships at a fitness center in m months is represented by $142 + 12m$. The number of family memberships at the fitness center in m months is represented by $52 + 6m$. Write a polynomial that represents the total number of memberships at the fitness center.

In Exercises 43–46, find the sum or difference.

43. $(2s^2 - 5st - t^2) - (s^2 + 7st - t^2)$

44. $(a^2 - 3ab + 2b^2) + (-4a^2 + 5ab - b^2)$

45. $(c^2 - 6d^2) + (c^2 - 2cd + 2d^2)$

46. $(-x^2 + 9xy) - (x^2 + 6xy - 8y^2)$

REASONING In Exercises 47–50, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.

47. The terms of a polynomial are ________ monomials.

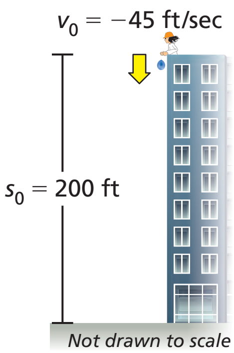
48. The difference of two trinomials is ________ a trinomial.

49. A binomial is ________ a polynomial of degree 2.

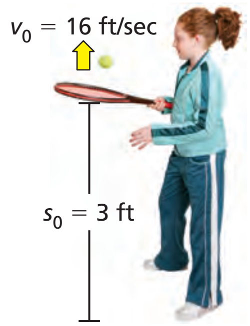
50. The sum of two polynomials is ________ a polynomial.

MODELING WITH MATHEMATICS The polynomial $-16t^2 + v_0t + s_0$ represents the height (in feet) of an object, where v_0 is the initial vertical velocity (in feet per second), s_0 is the initial height of the object (in feet), and t is the time (in seconds). In Exercises 51 and 52, write a polynomial that represents the height of the object. Then find the height of the object after 1 second.

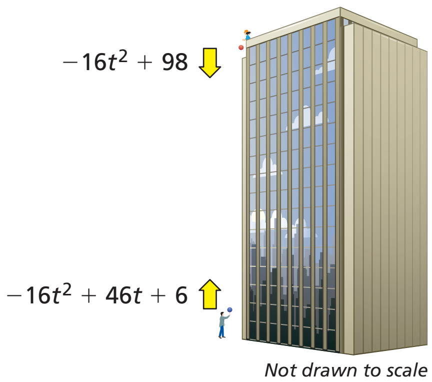
51. You throw a water balloon from a building.



52. You bounce a tennis ball on a racket.



53. **MODELING WITH MATHEMATICS** You drop a ball from a height of 98 feet. At the same time, your friend throws a ball upward. The polynomials represent the heights (in feet) of the balls after t seconds. *(See Example 6.)*



a. Before the balls reach the same height, write a polynomial that represents the distance between your ball and your friend's ball after t seconds.

b. Interpret the coefficients of the polynomial in part (a).

54. **MODELING WITH MATHEMATICS** During a 7-year period, the amounts (in millions of dollars) spent each year on buying new vehicles N and used vehicles U by United States residents are modeled by the equations

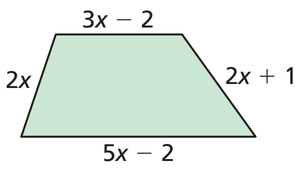
$$N = -0.028t^3 + 0.06t^2 + 0.1t + 17$$
$$U = -0.38t^2 + 1.5t + 42$$

where $t = 1$ represents the first year in the 7-year period.

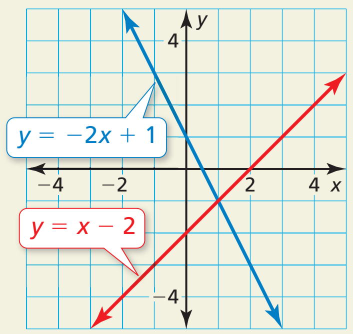
a. Write a polynomial that represents the total amount spent each year on buying new and used vehicles in the 7-year period.

b. How much is spent on buying new and used vehicles in the fifth year?

55. **MATHEMATICAL CONNECTIONS** Write the polynomial in standard form that represents the perimeter of the quadrilateral.



56. **HOW DO YOU SEE IT?** The right side of the equation of each line is a polynomial.



a. The absolute value of the difference of the two polynomials represents the vertical distance between points on the lines with the same x-value. Write this expression.

b. When does the expression in part (a) equal 0? How does this value relate to the graph?

57. **MAKING AN ARGUMENT** Your friend says that when adding polynomials, the order in which you add does not matter. Is your friend correct? Explain.

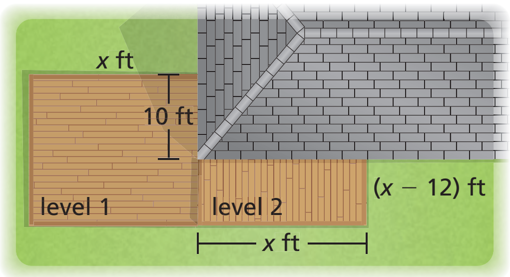
58. **THOUGHT PROVOKING** Write two polynomials whose sum is x^2 and whose difference is 1.

59. **REASONING** Determine whether the set is closed under the given operation. Explain.

a. the set of negative integers; multiplication

b. the set of whole numbers; addition

60. **PROBLEM SOLVING** You are building a multi-level deck.

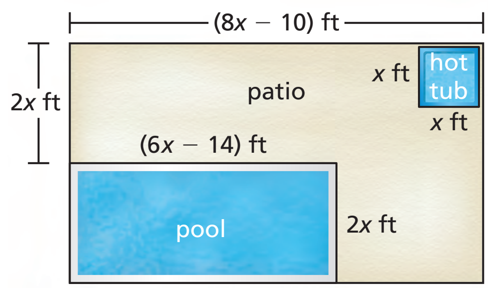


a. For each level, write a polynomial in standard form that represents the area of that level. Then write the polynomial in standard form that represents the total area of the deck.

b. What is the total area of the deck when $x = 20$?

c. A gallon of deck sealant covers 400 square feet. How many gallons of sealant do you need to cover the deck in part (b) once? Explain.

61. **PROBLEM SOLVING** A hotel installs a new swimming pool and a new hot tub.



a. Write the polynomial in standard form that represents the area of the patio.

b. The patio will cost \$10 per square foot. Determine the cost of the patio when $x = 9$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

62. $2(x - 1) + 3(x + 2)$

63. $8(4y - 3) + 2(y - 5)$

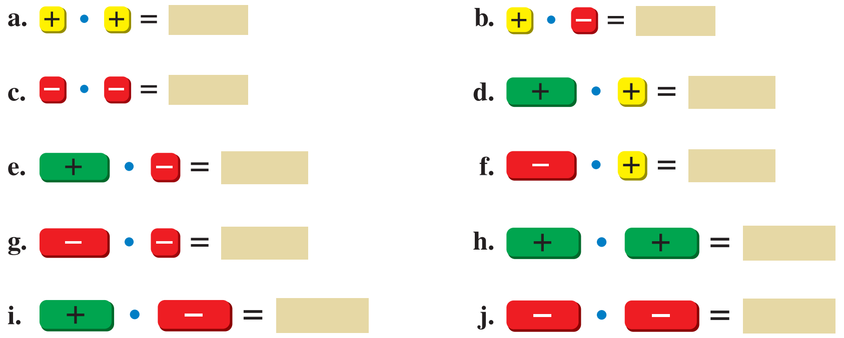
64. $5(2r + 1) - 3(-4r + 2)$

2.2 Multiplying Polynomials

Essential Question How can you multiply two polynomials?

EXPLORATION 1 **Multiplying Monomials Using Algebra Tiles**

Work with a partner. Write each product. Explain your reasoning.



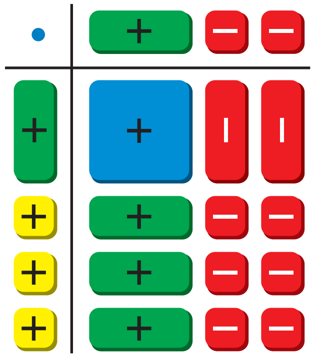
REASONING ABSTRACTLY

To be proficient in math, you need to reason abstractly and quantitatively. You need to pause as needed to recall the meanings of the symbols, operations, and quantities involved.

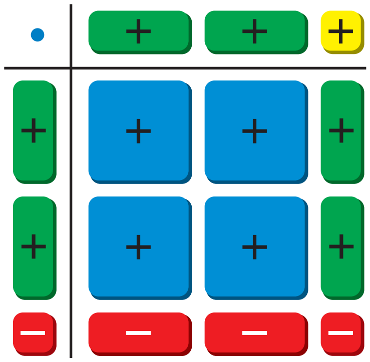
EXPLORATION 2 **Multiplying Binomials Using Algebra Tiles**

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles. In parts (c) and (d), first draw the rectangular array of algebra tiles that models each product.

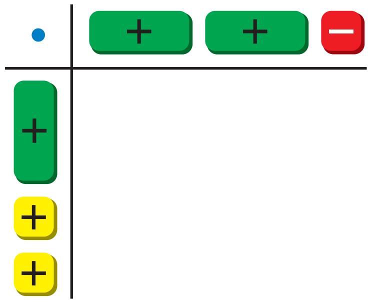
a. $(x + 3)(x - 2) =$



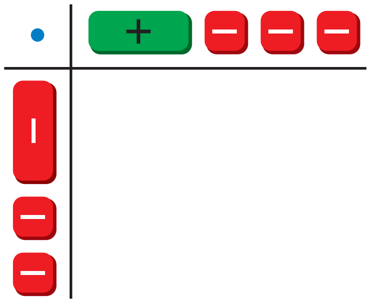
b. $(2x - 1)(2x + 1) =$



c. $(x + 2)(2x - 1) =$



d. $(-x - 2)(x - 3) =$



Communicate Your Answer

3. How can you multiply two polynomials?

4. Give another example of multiplying two binomials using algebra tiles that is similar to those in Exploration 2.

2.2 Lesson

Core Vocabulary

FOIL Method, *p. 71*

Previous
polynomial
closed
binomial
trinomial

What You Will Learn

- Multiply binomials.
- Use the FOIL Method.
- Multiply binomials and trinomials.

Multiplying Binomials

The product of two polynomials is always a polynomial. So, like the set of integers, the set of polynomials is closed under multiplication. You can use the Distributive Property to multiply two binomials.

EXAMPLE 1 Multiplying Binomials Using the Distributive Property

Find (a) $(x + 2)(x + 5)$ and (b) $(x + 3)(x - 4)$.

SOLUTION

a. Use the horizontal method.

$$(x + 2)(x + 5) = x(x + 5) + 2(x + 5) \qquad \text{Distribute } (x + 5) \text{ to each term of } (x + 2).$$

$$= x(x) + x(5) + 2(x) + 2(5) \qquad \text{Distributive Property}$$

$$= x^2 + 5x + 2x + 10 \qquad \text{Multiply.}$$

$$= x^2 + 7x + 10 \qquad \text{Combine like terms.}$$

▶ The product is $x^2 + 7x + 10$.

b. Use the vertical method.

$$\begin{array}{rrl}
 & x + 3 & \\
\times & x - 4 & \text{Align like terms vertically.} \\
\hline
 & -4x - 12 & \text{Distributive Property} \\
x^2 + 3x & & \text{Distributive Property} \\
\hline
x^2 - x & - 12 & \text{Combine like terms.}
\end{array}$$

Multiply $-4(x + 3)$.

Multiply $x(x + 3)$.

▶ The product is $x^2 - x - 12$.

EXAMPLE 2 Multiplying Binomials Using a Table

Find $(2x - 3)(x + 5)$.

SOLUTION

Step 1 Write each binomial as a sum of terms.

$$(2x - 3)(x + 5) = [2x + (-3)](x + 5)$$

Step 2 Make a table of products.

	$2x$	-3
x	$2x^2$	$-3x$
5	$10x$	-15

▶ The product is $2x^2 - 3x + 10x - 15$, or $2x^2 + 7x - 15$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the Distributive Property to find the product.

1. $(y + 4)(y + 1)$

2. $(z - 2)(z + 6)$

Use a table to find the product.

3. $(p + 3)(p - 8)$

4. $(r - 5)(2r - 1)$

Using the FOIL Method

The **FOIL Method** is a shortcut for multiplying two binomials.

Core Concept

FOIL Method

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms,	$(x+1)(x+2)$	$x(x) = x^2$
Outer terms,	$(x+1)(x+2)$	$x(2) = 2x$
Inner terms, and	$(x+1)(x+2)$	$1(x) = x$
Last terms.	$(x+1)(x+2)$	$1(2) = 2$

$$(x+1)(x+2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

EXAMPLE 3 Multiplying Binomials Using the FOIL Method

Find each product.

a. $(x-3)(x-6)$

b. $(2x+1)(3x-5)$

SOLUTION

a. Use the FOIL Method.

First Outer Inner Last

$$(x-3)(x-6) = x(x) + x(-6) + (-3)(x) + (-3)(-6)$$ FOIL Method

$$= x^2 + (-6x) + (-3x) + 18$$ Multiply.

$$= x^2 - 9x + 18$$ Combine like terms.

▶ The product is $x^2 - 9x + 18$.

b. Use the FOIL Method.

First Outer Inner Last

$$(2x+1)(3x-5) = 2x(3x) + 2x(-5) + 1(3x) + 1(-5)$$ FOIL Method

$$= 6x^2 + (-10x) + 3x + (-5)$$ Multiply.

$$= 6x^2 - 7x - 5$$ Combine like terms.

▶ The product is $6x^2 - 7x - 5$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the FOIL Method to find the product.

5. $(m-3)(m-7)$

6. $(x-4)(x+2)$

7. $\left(2u+\frac{1}{2}\right)\left(u-\frac{3}{2}\right)$

8. $(n+2)(n^2+3)$

Multiplying Binomials and Trinomials

EXAMPLE 4 Multiplying a Binomial and a Trinomial

Find $(x + 5)(x^2 - 3x - 2)$.

SOLUTION

$$\begin{array}{rrrrl}
 & x^2 & -3x & -2 & \\
\times & & x & +5 & \text{Align like terms vertically.} \\
\hline
 & 5x^2 & -15x & -10 & \text{Distributive Property} \\
x^3 & -3x^2 & -2x & & \text{Distributive Property} \\
\hline
x^3 & +2x^2 & -17x & -10 & \text{Combine like terms.}
\end{array}$$

Multiply $5(x^2 - 3x - 2)$.

Multiply $x(x^2 - 3x - 2)$.

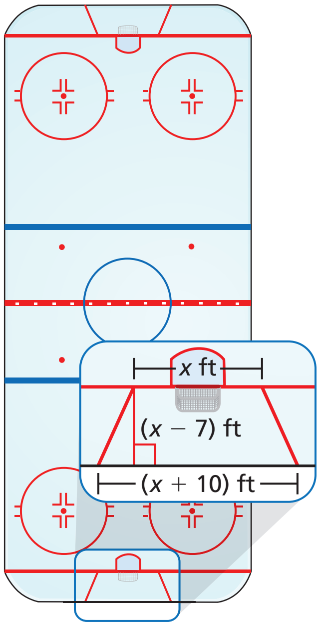
▶ The product is $x^3 + 2x^2 - 17x - 10$.

EXAMPLE 5 Solving a Real-Life Problem

In hockey, a goalie behind the goal line can only play a puck in the trapezoidal region.

a. Write a polynomial that represents the area of the trapezoidal region.

b. Find the area of the trapezoidal region when the shorter base is 18 feet.



SOLUTION

a. $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(x - 7)[x + (x + 10)]$

$= \frac{1}{2}(x - 7)(2x + 10)$

F O I L

$= \frac{1}{2}[2x^2 + 10x + (-14x) + (-70)]$

$= \frac{1}{2}(2x^2 - 4x - 70)$

$= x^2 - 2x - 35$

▶ A polynomial that represents the area of the trapezoidal region is $x^2 - 2x - 35$.

b. Find the value of $x^2 - 2x - 35$ when $x = 18$.

$x^2 - 2x - 35 = 18^2 - 2(18) - 35$ Substitute 18 for x.

$= 324 - 36 - 35$ Simplify.

$= 253$ Subtract.

▶ The area of the trapezoidal region is 253 square feet.

CONNECTIONS TO GEOMETRY

Recall that the formula for the area A of a trapezoid with height h and bases b_1 and b_2 is

$A = \frac{1}{2}h(b_1 + b_2)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the product.

9. $(x + 1)(x^2 + 5x + 8)$

10. $(n - 3)(n^2 - 2n + 4)$

11. **WHAT IF?** In Example 5(a), how does the polynomial change when the longer base is extended by 1 foot? Explain.

2.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Describe two ways to find the product of two binomials.

2. **WRITING** Explain how the letters of the word FOIL can help you to remember how to multiply two binomials.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, use the Distributive Property to find the product. *(See Example 1.)*

3. $(x + 1)(x + 3)$
4. $(y + 6)(y + 4)$
5. $(z - 5)(z + 3)$
6. $(a + 8)(a - 3)$
7. $(g - 7)(g - 2)$
8. $(n - 6)(n - 4)$
9. $(3m + 1)(m + 9)$
10. $(5s + 6)(s - 2)$

In Exercises 11–18, use a table to find the product. *(See Example 2.)*

11. $(x + 3)(x + 2)$
12. $(y + 10)(y - 5)$
13. $(h - 8)(h - 9)$
14. $(c - 6)(c - 5)$
15. $(3k - 1)(4k + 9)$
16. $(5g + 3)(g + 8)$
17. $(-3 + 2j)(4j - 7)$
18. $(5d - 12)(-7 + 3d)$

ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in finding the product of the binomials.

19. ✗

$$\begin{aligned}(t-2)(t+5) &= t-2(t+5)\\ &= t-2t-10\\ &= -t-10\end{aligned}$$

20. ✗

$(x-5)(3x+1)$

	$3x$	1
x	$3x^2$	x
5	$15x$	5

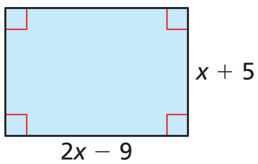
$(x-5)(3x+1) = 3x^2 + 16x + 5$

In Exercises 21–30, use the FOIL Method to find the product. *(See Example 3.)*

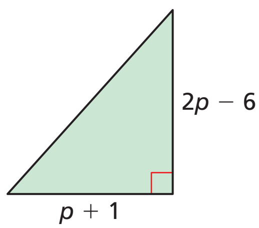
21. $(b + 3)(b + 7)$
22. $(w + 9)(w + 6)$
23. $(k + 5)(k - 1)$
24. $(x - 4)(x + 8)$
25. $\left(q - \frac{3}{4}\right)\left(q + \frac{1}{4}\right)$
26. $\left(z - \frac{5}{3}\right)\left(z - \frac{2}{3}\right)$
27. $(9 - r)(2 - 3r)$
28. $(8 - 4x)(2x + 6)$
29. $(w + 5)(w^2 + 3w)$
30. $(v - 3)(v^2 + 8v)$

MATHEMATICAL CONNECTIONS In Exercises 31–34, write a polynomial that represents the area of the shaded region.

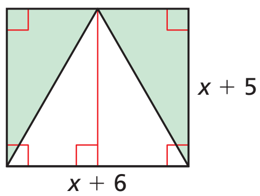
31.



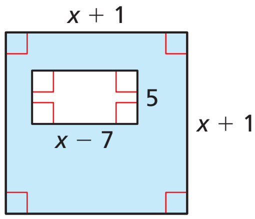
32.



33.



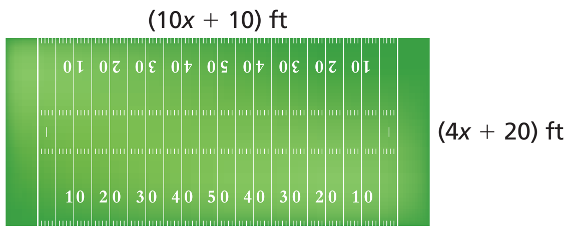
34.



In Exercises 35–42, find the product. *(See Example 4.)*

35. $(x + 4)(x^2 + 3x + 2)$
36. $(f + 1)(f^2 + 4f + 8)$
37. $(y + 3)(y^2 + 8y - 2)$
38. $(t - 2)(t^2 - 5t + 1)$
39. $(4 - b)(5b^2 + 5b - 4)$
40. $(d + 6)(2d^2 - d + 7)$
41. $(3e^2 - 5e + 7)(6e + 1)$
42. $(6v^2 + 2v - 9)(4 - 5v)$

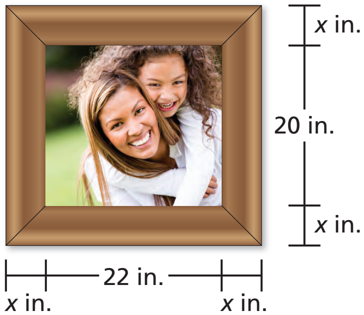
43. MODELING WITH MATHEMATICS The football field is rectangular. *(See Example 5.)*



a. Write a polynomial that represents the area of the football field.

b. Find the area of the football field when the width is 160 feet.

44. MODELING WITH MATHEMATICS You design a frame to surround a rectangular photo. The width of the frame is the same on every side, as shown.



a. Write a polynomial that represents the combined area of the photo and the frame.

b. Find the combined area of the photo and the frame when the width of the frame is 4 inches.

45. WRITING When multiplying two binomials, explain how the degree of the product is related to the degree of each binomial.

46. THOUGHT PROVOKING Write two polynomials that are not monomials whose product is a trinomial of degree 3.

47. MAKING AN ARGUMENT Your friend says the FOIL Method can be used to multiply two trinomials. Is your friend correct? Explain your reasoning.

48. HOW DO YOU SEE IT? The table shows one method of finding the product of two binomials.

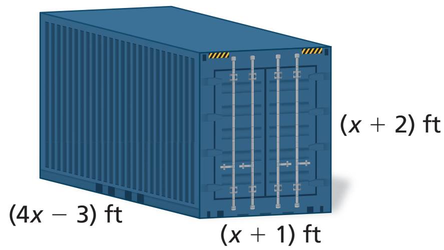
	$-4x$	3
$-8x$	a	b
-9	c	d

a. Write the two binomials being multiplied.

b. Determine whether a, b, c, and d will be positive or negative when $x > 0$.

49. COMPARING METHODS You use the Distributive Property to multiply $(x + 3)(x - 5)$. Your friend uses the FOIL Method to multiply $(x - 5)(x + 3)$. Should your answers be equivalent? Justify your answer.

50. USING STRUCTURE The shipping container is a rectangular prism. Write a polynomial that represents the volume of the container.



51. ABSTRACT REASONING The product of $(x + m)(x + n)$ is $x^2 + bx + c$.

a. What do you know about m and n when $c > 0$?

b. What do you know about m and n when $c < 0$?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the absolute value function as a piecewise function. *(Section 1.2)*

52. $y = |x| + 4$

53. $y = 6|x - 3|$

54. $y = -4|x + 2|$

Simplify the expression. Write your answer using only positive exponents. *(Section 1.4)*

55. $10^2 \cdot 10^9$

56. $\dfrac{x^5 \cdot x}{x^8}$

57. $(3z^6)^{-3}$

58. $\left(\dfrac{2y^4}{y^3}\right)^{-2}$

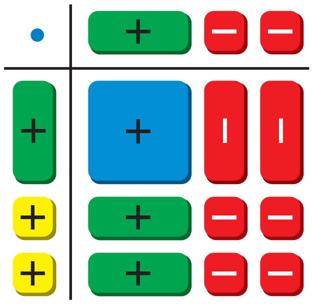
2.3 Special Products of Polynomials

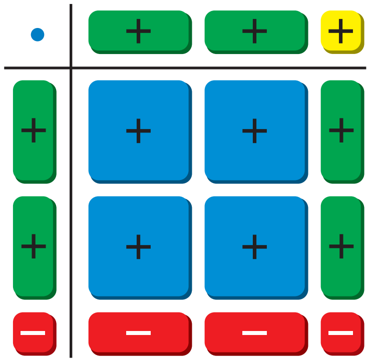
Essential Question What are the patterns in the special products $(a+b)(a-b)$, $(a+b)^2$, and $(a-b)^2$?

EXPLORATION 1 Finding a Sum and Difference Pattern

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles.

a. $(x+2)(x-2) =$ ______ **b.** $(2x-1)(2x+1) =$ ______

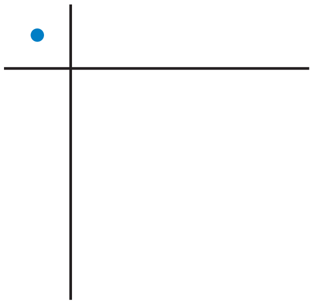


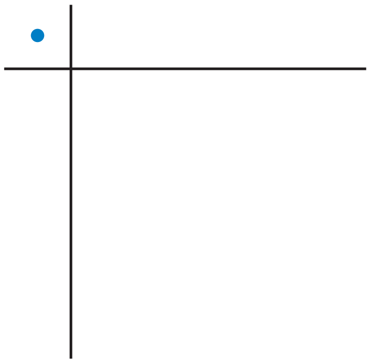


EXPLORATION 2 Finding the Square of a Binomial Pattern

Work with a partner. Draw the rectangular array of algebra tiles that models each product of two binomials. Write the product.

a. $(x+2)^2 =$ ______ **b.** $(2x-1)^2 =$ ______





LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. What are the patterns in the special products $(a+b)(a-b)$, $(a+b)^2$, and $(a-b)^2$?

4. Use the appropriate special product pattern to find each product. Check your answers using algebra tiles.

a. $(x+3)(x-3)$ **b.** $(x-4)(x+4)$ **c.** $(3x+1)(3x-1)$

d. $(x+3)^2$ **e.** $(x-2)^2$ **f.** $(3x+1)^2$

2.3 Lesson

Core Vocabulary

Previous
binomial

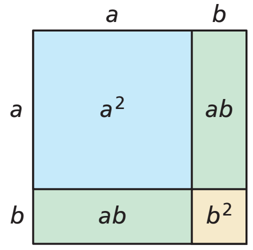
What You Will Learn

- Use the square of a binomial pattern.
- Use the sum and difference pattern.
- Use special product patterns to solve real-life problems.

Using the Square of a Binomial Pattern

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$ — Replace b with $-b$ in the pattern above.

$(a - b)^2 = a^2 - 2ab + b^2$ — Simplify.

Core Concept

Square of a Binomial Pattern

Algebra

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Example

$$(x + 5)^2 = (x)^2 + 2(x)(5) + (5)^2 = x^2 + 10x + 25$$

$$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2 = 4x^2 - 12x + 9$$

LOOKING FOR STRUCTURE

When you use special product patterns, remember that a and b can be numbers, variables, or variable expressions.

EXAMPLE 1 Using the Square of a Binomial Pattern

Find each product.

a. $(3x + 4)^2$ **b.** $(5x - 2y)^2$

SOLUTION

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ — Square of a binomial pattern

$= 9x^2 + 24x + 16$ — Simplify.

▶ The product is $9x^2 + 24x + 16$.

b. $(5x - 2y)^2 = (5x)^2 - 2(5x)(2y) + (2y)^2$ — Square of a binomial pattern

$= 25x^2 - 20xy + 4y^2$ — Simplify.

▶ The product is $25x^2 - 20xy + 4y^2$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the product.

1. $(x + 7)^2$ **2.** $(7x - 3)^2$ **3.** $(4x - y)^2$ **4.** $(3m + n)^2$

Using the Sum and Difference Pattern

To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL Method.

$$(x + 2)(x - 2) = x^2 - 2x + 2x - 4 \qquad \text{FOIL Method}$$
$$= x^2 - 4 \qquad \text{Combine like terms.}$$

This suggests a pattern for the product of the sum and difference of two terms.

Core Concept

Sum and Difference Pattern

Algebra	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 Using the Sum and Difference Pattern

Find each product.

a. $(t + 5)(t - 5)$

b. $(3x + y)(3x - y)$

SOLUTION

a. $(t + 5)(t - 5) = t^2 - 5^2$ Sum and difference pattern

$= t^2 - 25$ Simplify.

▶ The product is $t^2 - 25$.

b. $(3x + y)(3x - y) = (3x)^2 - y^2$ Sum and difference pattern

$= 9x^2 - y^2$ Simplify.

▶ The product is $9x^2 - y^2$.

The special product patterns can help you use mental math to find certain products of numbers.

EXAMPLE 3 Using Special Product Patterns and Mental Math

Use special product patterns to find the product $26 \cdot 34$.

SOLUTION

Notice that 26 is 4 less than 30, while 34 is 4 more than 30.

$26 \cdot 34 = (30 - 4)(30 + 4)$ Write as product of difference and sum.

$= 30^2 - 4^2$ Sum and difference pattern

$= 900 - 16$ Evaluate powers.

$= 884$ Simplify.

▶ The product is 884.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

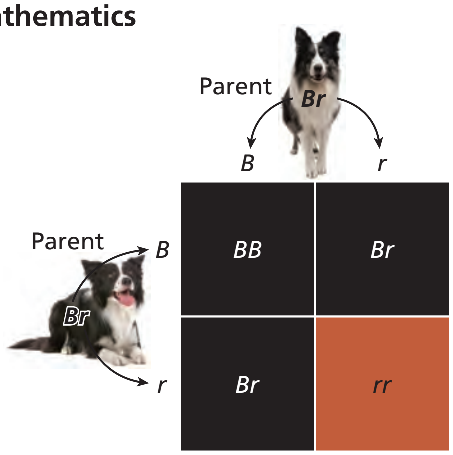
Find the product.

5. $(x + 10)(x - 10)$ **6.** $(2x + 1)(2x - 1)$ **7.** $(x + 3y)(x - 3y)$

8. Describe how to use special product patterns to find 21^2.

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics



A combination of two genes determines the color of the dark patches of a border collie's coat. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene B is for black patches, and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch colors.

a. What percent of the possible gene combinations result in black patches?

b. Show how you could use a polynomial to model the possible gene combinations.

SOLUTION

a. Notice that the Punnett square shows four possible gene combinations of the offspring. Of these combinations, three result in black patches.

▶ So, 75% of the possible gene combinations result in black patches.

b. Model the gene from each parent with $0.5B + 0.5r$. There is an equal chance that the offspring inherits a black or a red gene from each parent.

You can model the possible gene combinations of the offspring with $(0.5B + 0.5r)^2$. Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

$$(0.5B + 0.5r)^2 = (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2$$
$$= 0.25B^2 + 0.5Br + 0.25r^2$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

25% BB, black patches

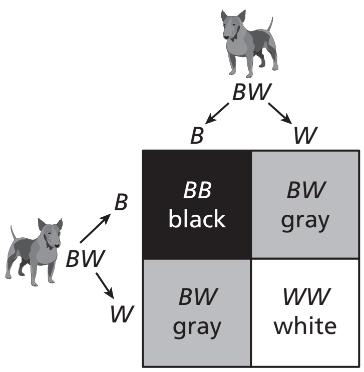
50% Br, black patches

25% rr, red patches

The coefficients show that 25% + 50% = 75% of the possible gene combinations result in black patches.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*



9. Each of two dogs has one black gene (B) and one white gene (W). The Punnett square shows the possible gene combinations of an offspring and the resulting colors.

a. What percent of the possible gene combinations result in black?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.

2.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. WRITING Explain how to use the square of a binomial pattern.

2. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$(x + 1)(x - 1)$	$(3x + 2)(3x - 2)$	$(x + 2)(x - 3)$	$(2x + 5)(2x - 5)$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, find the product. *(See Example 1.)*

3. $(x + 8)^2$

4. $(a - 6)^2$

5. $(2f - 1)^2$

6. $(5p + 2)^2$

7. $(-7t + 4)^2$

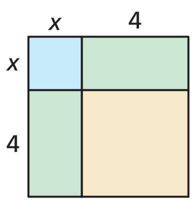
8. $(-12 - n)^2$

9. $(2a + b)^2$

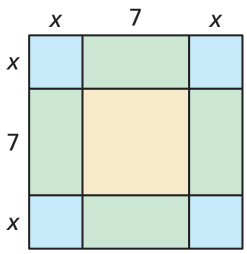
10. $(6x - 3y)^2$

MATHEMATICAL CONNECTIONS **In Exercises 11–14, write a polynomial that represents the area of the square.**

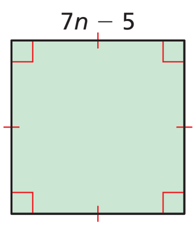
11.



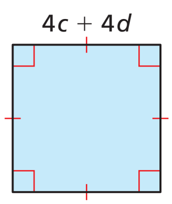
12.



13.



14.



In Exercises 15–24, find the product. *(See Example 2.)*

15. $(t - 7)(t + 7)$

16. $(m + 6)(m - 6)$

17. $(4x + 1)(4x - 1)$

18. $(2k - 4)(2k + 4)$

19. $(8 + 3a)(8 - 3a)$

20. $\left(\frac{1}{2} - c\right)\left(\frac{1}{2} + c\right)$

21. $(p - 10q)(p + 10q)$

22. $(7m + 8n)(7m - 8n)$

23. $(-y + 4)(-y - 4)$

24. $(-5g - 2h)(-5g + 2h)$

In Exercises 25–30, use special product patterns to find the product. *(See Example 3.)*

25. $16 \cdot 24$

26. $33 \cdot 27$

27. 42^2

28. 29^2

29. 30.5^2

30. $10\frac{1}{3} \cdot 9\frac{2}{3}$

ERROR ANALYSIS **In Exercises 31 and 32, describe and correct the error in finding the product.**

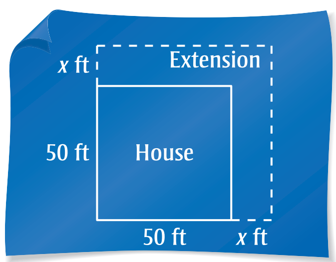
31.

$$\begin{aligned}(k + 4)^2 &= k^2 + 4^2 \\ &= k^2 + 16\end{aligned}$$

32.

$$\begin{aligned}(s + 5)(s - 5) &= s^2 + 2(s)(5) - 5^2 \\ &= s^2 + 10s - 25\end{aligned}$$

33. MODELING WITH MATHEMATICS A contractor extends a house on two sides.



a. The area of the house after the renovation is represented by $(x + 50)^2$. Find this product.

b. Use the polynomial in part (a) to find the area when $x = 15$. What is the area of the extension?

34. **MODELING WITH MATHEMATICS** A square-shaped parking lot with 100-foot sides is reduced by x feet on one side and extended by x feet on an adjacent side.

a. The area of the new parking lot is represented by $(100 - x)(100 + x)$. Find this product.

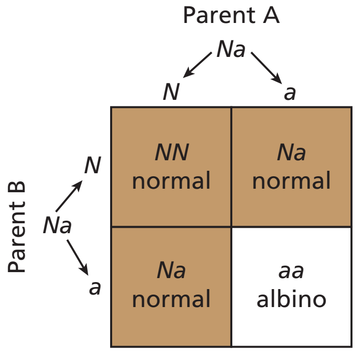
b. Does the area of the parking lot increase, decrease, or stay the same? Explain.

c. Use the polynomial in part (a) to find the area of the new parking lot when $x = 21$.

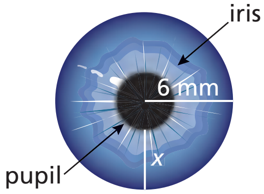
35. **MODELING WITH MATHEMATICS** In deer, the gene N is for normal coloring and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. The Punnett square shows the possible gene combinations of an offspring and the resulting colors from parents that both have the gene combination Na. *(See Example 4.)*

a. What percent of the possible gene combinations result in albino coloring?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.



36. **MODELING WITH MATHEMATICS** Your iris controls the amount of light that enters your eye by changing the size of your pupil.

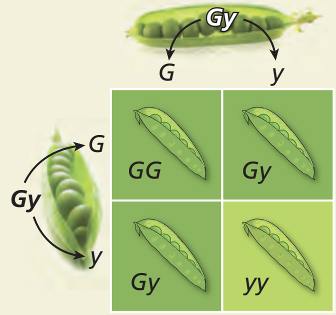


a. Write a polynomial that represents the area of your pupil. Write your answer in terms of π.

b. The width x of your iris decreases from 4 millimeters to 2 millimeters when you enter a dark room. How many times greater is the area of your pupil after entering the room than before entering the room? Explain.

37. **CRITICAL THINKING** Write two binomials that have the product $x^2 - 121$. Explain.

38. **HOW DO YOU SEE IT?** In pea plants, any gene combination with a green gene (G) results in a green pod. The Punnett square shows the possible gene combinations of the offspring of two Gy pea plants and the resulting pod colors.



G = Green gene
y = Yellow gene

A polynomial that models the possible gene combinations of the offspring is

$$(0.5G + 0.5y)^2 = 0.25G^2 + 0.5Gy + 0.25y^2.$$

Describe two ways to determine the percent of possible gene combinations that result in green pods.

In Exercises 39–42, find the product.

39. $(x^2 + 1)(x^2 - 1)$

40. $(y^3 + 4)^2$

41. $(2m^2 - 5n^2)^2$

42. $(r^3 - 6t^4)(r^3 + 6t^4)$

43. **MAKING AN ARGUMENT** Your friend claims to be able to use a special product pattern to determine that $\left(4\frac{1}{3}\right)^2$ is equal to $16\frac{1}{9}$. Is your friend correct? Explain.

44. **THOUGHT PROVOKING** Modify the dimensions of the original parking lot in Exercise 34 so that the area can be represented by two other types of special product patterns discussed in this section. Is there a positive x-value for which the three area expressions are equivalent? Explain.

45. **REASONING** Find k so that $9x^2 - 48x + k$ is the square of a binomial.

46. **REPEATED REASONING** Find $(x + 1)^3$ and $(x + 2)^3$. Find a pattern in the terms and use it to write a pattern for the cube of a binomial $(a + b)^3$.

47. **PROBLEM SOLVING** Find two numbers a and b such that $(a + b)(a - b) < (a - b)^2 < (a + b)^2$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the expression using the GCF. *(Skills Review Handbook)*

48. $12y - 18$

49. $9r + 27$

50. $49s + 35t$

51. $15x - 10y$

2.4 Solving Polynomial Equations in Factored Form

Essential Question

How can you solve a polynomial equation?

EXPLORATION 1 Matching Equivalent Forms of an Equation

Work with a partner. An equation is considered to be in *factored form* when the product of the factors is equal to 0. Match each factored form of the equation with its equivalent standard form and nonstandard form.

	Factored Form		Standard Form		Nonstandard Form
a.	$(x-1)(x-3)=0$	**A.**	$x^2-x-2=0$	**1.**	$x^2-5x=-6$
b.	$(x-2)(x-3)=0$	**B.**	$x^2+x-2=0$	**2.**	$(x-1)^2=4$
c.	$(x+1)(x-2)=0$	**C.**	$x^2-4x+3=0$	**3.**	$x^2-x=2$
d.	$(x-1)(x+2)=0$	**D.**	$x^2-5x+6=0$	**4.**	$x(x+1)=2$
e.	$(x+1)(x-3)=0$	**E.**	$x^2-2x-3=0$	**5.**	$x^2-4x=-3$

USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider using tools such as a table or a spreadsheet to organize your results.

EXPLORATION 2 Writing a Conjecture

Work with a partner. Substitute 1, 2, 3, 4, 5, and 6 for x in each equation and determine whether the equation is true. Organize your results in a table. Write a conjecture describing what you discovered.

a. $(x-1)(x-2)=0$

b. $(x-2)(x-3)=0$

c. $(x-3)(x-4)=0$

d. $(x-4)(x-5)=0$

e. $(x-5)(x-6)=0$

f. $(x-6)(x-1)=0$

EXPLORATION 3 Special Properties of 0 and 1

Work with a partner. The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

a. When you add ____ to a number n, you get n.

b. If the product of two numbers is ____, then at least one of the numbers is 0.

c. The square of ____ is equal to itself.

d. When you multiply a number n by ____, you get n.

e. When you multiply a number n by ____, you get 0.

f. The opposite of ____ is equal to itself.

Communicate Your Answer

4. How can you solve a polynomial equation?

5. One of the properties in Exploration 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Why do you think it is called the Zero-Product Property? Explain how it is used in algebra and why it is so important.

2.4 Lesson

What You Will Learn

- Use the Zero-Product Property.
- Factor polynomials using the GCF.
- Use the Zero-Product Property to solve real-life problems.

Core Vocabulary

factored form, *p. 82*
Zero-Product Property, *p. 82*
roots, *p. 82*
repeated roots, *p. 83*

Previous
polynomial
standard form
greatest common factor (GCF)
monomial

Using the Zero-Product Property

A polynomial is in **factored form** when it is written as a product of factors.

Standard form	Factored form
$x^2 + 2x$	$x(x + 2)$
$x^2 + 5x - 24$	$(x - 3)(x + 8)$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots**.

Core Concept

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $2x(x - 4) = 0$ **b.** $(x - 3)(x - 9) = 0$

SOLUTION

a. $2x(x - 4) = 0$ Write equation.

$2x = 0 \quad or \quad x - 4 = 0$ Zero-Product Property

$x = 0 \quad or \quad x = 4$ Solve for x.

▶ The roots are $x = 0$ and $x = 4$.

b. $(x - 3)(x - 9) = 0$ Write equation.

$x - 3 = 0 \quad or \quad x - 9 = 0$ Zero-Product Property

$x = 3 \quad or \quad x = 9$ Solve for x.

▶ The roots are $x = 3$ and $x = 9$.

Check

To check the solutions of Example 1(a), substitute each solution in the original equation.

$2(0)(0 - 4) \stackrel{?}{=} 0$

$0(-4) \stackrel{?}{=} 0$

$0 = 0$ ✓

$2(4)(4 - 4) \stackrel{?}{=} 0$

$8(0) \stackrel{?}{=} 0$

$0 = 0$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Check your solutions.

1. $x(x - 1) = 0$
2. $3t(t + 2) = 0$
3. $(z - 4)(z - 6) = 0$

When two or more roots of an equation are the same number, the equation has **repeated roots**.

EXAMPLE 2 Solving Polynomial Equations

Solve each equation.

a. $(2x + 7)(2x - 7) = 0$ **b.** $(x - 1)^2 = 0$ **c.** $(x + 1)(x - 3)(x - 2) = 0$

SOLUTION

a. $(2x + 7)(2x - 7) = 0$ Write equation.

$2x + 7 = 0$ *or* $2x - 7 = 0$ Zero-Product Property

$x = -\frac{7}{2}$ *or* $x = \frac{7}{2}$ Solve for x.

▶ The roots are $x = -\frac{7}{2}$ and $x = \frac{7}{2}$.

b. $(x - 1)^2 = 0$ Write equation.

$(x - 1)(x - 1) = 0$ Expand equation.

$x - 1 = 0$ *or* $x - 1 = 0$ Zero-Product Property

$x = 1$ *or* $x = 1$ Solve for x.

▶ The equation has repeated roots of $x = 1$.

c. $(x + 1)(x - 3)(x - 2) = 0$ Write equation.

$x + 1 = 0$ *or* $x - 3 = 0$ *or* $x - 2 = 0$ Zero-Product Property

$x = -1$ *or* $x = 3$ *or* $x = 2$ Solve for x.

▶ The roots are $x = -1$, $x = 3$, and $x = 2$.

STUDY TIP

You can extend the Zero-Product Property to products of more than two real numbers.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Check your solutions.

4. $(3s + 5)(5s + 8) = 0$ **5.** $(b + 7)^2 = 0$ **6.** $(d - 2)(d + 6)(d + 8) = 0$

Factoring Polynomials Using the GCF

To solve a polynomial equation using the Zero-Product Property, you may need to *factor* the polynomial, or write it as a product of other polynomials. Look for the *greatest common factor* (GCF) of the terms of the polynomial. This is a monomial that divides evenly into each term.

EXAMPLE 3 Finding the Greatest Common Monomial Factor

Factor out the greatest common monomial factor from $4x^4 + 24x^3$.

SOLUTION

The GCF of 4 and 24 is 4. The GCF of x^4 and x^3 is x^3. So, the greatest common monomial factor of the terms is $4x^3$.

▶ So, $4x^4 + 24x^3 = 4x^3(x + 6)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. Factor out the greatest common monomial factor from $8y^2 - 24y$.

EXAMPLE 4 Solving Equations by Factoring

Solve (a) $2x^2 + 8x = 0$ and (b) $6n^2 = 15n$.

SOLUTION

a.

$2x^2 + 8x = 0$	Write equation.
$2x(x + 4) = 0$	Factor left side.
$2x = 0$ *or* $x + 4 = 0$	Zero-Product Property
$x = 0$ *or* $x = -4$	Solve for x.

▶ The roots are $x = 0$ and $x = -4$.

b.

$6n^2 = 15n$	Write equation.
$6n^2 - 15n = 0$	Subtract $15n$ from each side.
$3n(2n - 5) = 0$	Factor left side.
$3n = 0$ *or* $2n - 5 = 0$	Zero-Product Property
$n = 0$ *or* $n = \frac{5}{2}$	Solve for n.

▶ The roots are $n = 0$ and $n = \frac{5}{2}$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the equation. Check your solutions.

8. $a^2 + 5a = 0$

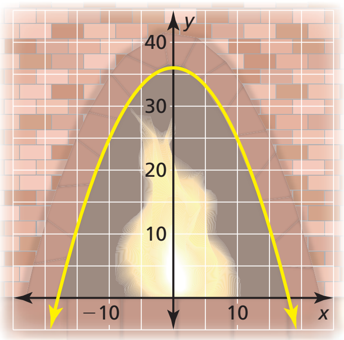
9. $3s^2 - 9s = 0$

10. $4x^2 = 2x$

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics

You can model the arch of a fireplace using the equation $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x-axis represents the floor. Find the width of the arch at floor level.



SOLUTION

Use the x-coordinates of the points where the arch meets the floor to find the width. At floor level, $y = 0$. So, substitute 0 for y and solve for x.

$y = -\frac{1}{9}(x + 18)(x - 18)$	Write equation.
$0 = -\frac{1}{9}(x + 18)(x - 18)$	Substitute 0 for y.
$0 = (x + 18)(x - 18)$	Multiply each side by -9.
$x + 18 = 0$ *or* $x - 18 = 0$	Zero-Product Property
$x = -18$ *or* $x = 18$	Solve for x.

The width is the distance between the x-coordinates, -18 and 18.

▶ So, the width of the arch at floor level is $|-18 - 18| = 36$ inches.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

11. You can model the entrance to a mine shaft using the equation $y = -\frac{1}{2}(x + 4)(x - 4)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at ground level.

2.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. WRITING Explain how to use the Zero-Product Property to find the solutions of the equation $3x(x - 6) = 0$.

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find *both* answers.

Solve the equation $(2k + 4)(k - 3) = 0$.	Find the values of k for which $2k + 4 = 0$ *or* $k - 3 = 0$.
Find the value of k for which $(2k + 4) + (k - 3) = 0$.	Find the roots of the equation $(2k + 4)(k - 3) = 0$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, solve the equation. *(See Example 1.)*

3. $x(x + 7) = 0$

4. $r(r - 10) = 0$

5. $12t(t - 5) = 0$

6. $-2v(v + 1) = 0$

7. $(s - 9)(s - 1) = 0$

8. $(y + 2)(y - 6) = 0$

In Exercises 9–20, solve the equation. *(See Example 2.)*

9. $(2a - 6)(3a + 15) = 0$

10. $(4q + 3)(q + 2) = 0$

11. $(5m + 4)^2 = 0$

12. $(h - 8)^2 = 0$

13. $(3 - 2g)(7 - g) = 0$

14. $(2 - 4d)(2 + 4d) = 0$

15. $z(z + 2)(z - 1) = 0$

16. $5p(2p - 3)(p + 7) = 0$

17. $(r - 4)^2(r + 8) = 0$

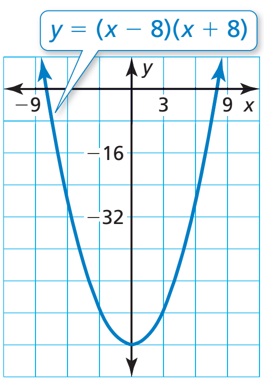
18. $w(w - 6)^2 = 0$

19. $(15 - 5c)(5c + 5)(-c + 6) = 0$

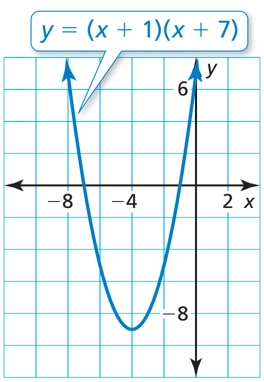
20. $(2 - n)\left(6 + \frac{2}{3}n\right)(n - 2) = 0$

In Exercises 21–24, find the x-coordinates of the points where the graph crosses the x-axis.

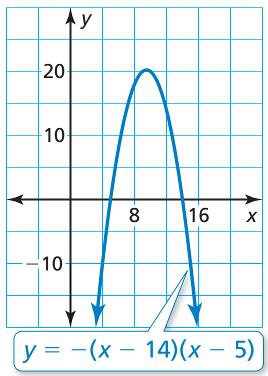
21.



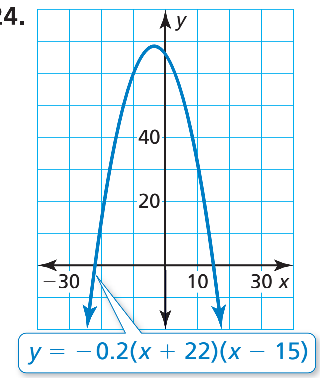
22.



23.



24.



In Exercises 25–30, factor the polynomial. *(See Example 3.)*

25. $5z^2 + 45z$

26. $6d^2 - 21d$

27. $3y^3 - 9y^2$

28. $20x^3 + 30x^2$

29. $5n^6 + 2n^5$

30. $12a^4 + 8a$

In Exercises 31–36, solve the equation. *(See Example 4.)*

31. $4p^2 - p = 0$

32. $6m^2 + 12m = 0$

33. $25c + 10c^2 = 0$

34. $18q - 2q^2 = 0$

35. $3n^2 = 9n$

36. $-28r = 4r^2$

37. ERROR ANALYSIS Describe and correct the error in solving the equation.

✗

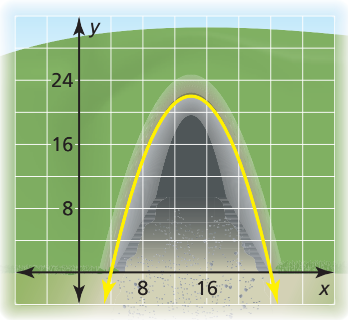
$$6x(x + 5) = 0$$
$$x + 5 = 0$$
$$x = -5$$

The root is $x = -5$.

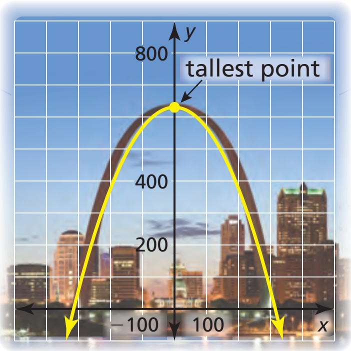
38. ERROR ANALYSIS Describe and correct the error in solving the equation.

✗
$$3y^2 = 21y$$
$$3y = 21$$
$$y = 7$$
The root is $y = 7$.

39. MODELING WITH MATHEMATICS The entrance of a tunnel can be modeled by $y = -\frac{11}{50}(x - 4)(x - 24)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the tunnel at ground level. *(See Example 5.)*



40. MODELING WITH MATHEMATICS The Gateway Arch in St. Louis can be modeled by $y = -\frac{2}{315}(x + 315)(x - 315)$, where x and y are measured in feet. The x-axis represents the ground.

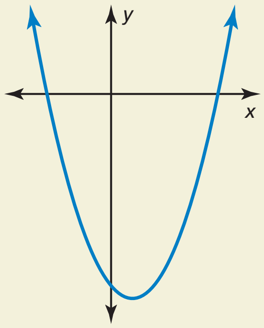


a. Find the width of the arch at ground level.

b. How tall is the arch?

41. MODELING WITH MATHEMATICS A penguin leaps out of the water while swimming. This action is called porpoising. The height y (in feet) of a porpoising penguin can be modeled by $y = -16x^2 + 4.8x$, where x is the time (in seconds) since the penguin leaped out of the water. Find the roots of the equation when $y = 0$. Explain what the roots mean in this situation.

42. HOW DO YOU SEE IT? Use the graph to fill in each blank in the equation with the symbol + or −. Explain your reasoning.



$y = (x \;\_\_\; 5)(x \;\_\_\; 3)$

43. CRITICAL THINKING How many x-intercepts does the graph of $y = (2x + 5)(x - 9)^2$ have? Explain.

44. MAKING AN ARGUMENT Your friend says that the graph of the equation $y = (x - a)(x - b)$ always has two x-intercepts for any values of a and b. Is your friend correct? Explain.

45. CRITICAL THINKING Does the equation $(x^2 + 3)(x^4 + 1) = 0$ have any real roots? Explain.

46. THOUGHT PROVOKING Write a polynomial equation of degree 4 whose only roots are $x = 1$, $x = 2$, and $x = 3$.

47. REASONING Find the values of x in terms of y that are solutions of each equation.

a. $(x + y)(2x - y) = 0$

b. $(x^2 - y^2)(4x + 16y) = 0$

48. PROBLEM SOLVING Solve the equation $(4^{x-5} - 16)(3^x - 81) = 0$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

List the factor pairs of the number. *(Skills Review Handbook)*

49. 10

50. 18

51. 30

52. 48

2.1–2.4 What Did You Learn?

Core Vocabulary

monomial, *p. 62*
degree of a monomial, *p. 62*
polynomial, *p. 63*
binomial, *p. 63*
trinomial, *p. 63*
degree of a polynomial, *p. 63*
standard form, *p. 63*
leading coefficient, *p. 63*
closed, *p. 64*
FOIL Method, *p. 71*
factored form, *p. 82*
Zero-Product Property, *p. 82*
roots, *p. 82*
repeated roots, *p. 83*

Core Concepts

Section 2.1

Polynomials, *p. 63*
Adding Polynomials, *p. 64*
Subtracting Polynomials, *p. 64*

Section 2.2

Multiplying Binomials, *p. 70*
FOIL Method, *p. 71*
Multiplying Binomials and Trinomials, *p. 72*

Section 2.3

Square of a Binomial Pattern, *p. 76*
Sum and Difference Pattern, *p. 77*

Section 2.4

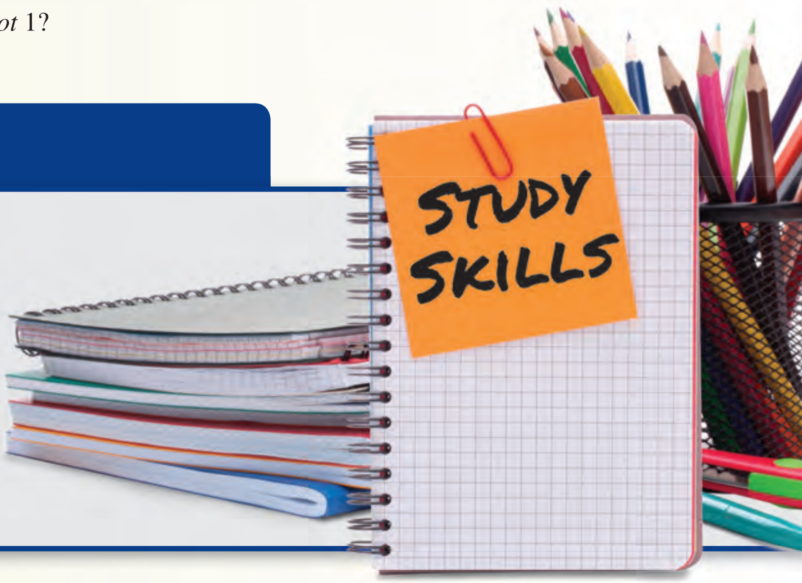
Zero-Product Property, *p. 82*
Factoring Polynomials Using the GCF, *p. 83*

Mathematical Practices

1. Explain how you wrote the polynomial in Exercise 11 on page 79. Is there another method you can use to write the same polynomial?
2. Find a shortcut for exercises like Exercise 7 on page 85 when the variable has a coefficient of 1. Does your shortcut work when the coefficient is *not* 1?

Preparing for a Test

- Review examples of each type of problem that could appear on the test. Use the tutorials at *BigIdeasMath.com* for additional help.
- Review the homework problems your teacher assigned.
- Take a practice test.



2.1–2.4 Quiz

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms. *(Section 2.1)*

1. $-8q^3$

2. $9 + d^2 - 3d$

3. $\frac{2}{3}m^4 - \frac{5}{6}m^6$

4. $-1.3z + 3z^4 + 7.4z^2$

Find the sum or difference. *(Section 2.1)*

5. $(2x^2 + 5) + (-x^2 + 4)$

6. $(-3n^2 + n) - (2n^2 - 7)$

7. $(-p^2 + 4p) - (p^2 - 3p + 15)$

8. $(a^2 - 3ab + b^2) + (-a^2 + ab + b^2)$

Find the product. *(Section 2.2 and Section 2.3)*

9. $(w + 6)(w + 7)$

10. $(3 - 4d)(2d - 5)$

11. $(y + 9)(y^2 + 2y - 3)$

12. $(3z - 5)(3z + 5)$

13. $(t + 5)^2$

14. $(2q - 6)^2$

Solve the equation. *(Section 2.4)*

15. $5x^2 - 15x = 0$

16. $(8 - g)(8 - g) = 0$

17. $(3p + 7)(3p - 7)(p + 8) = 0$

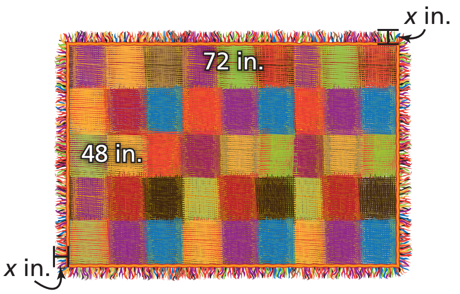
18. $-3y(y - 8)(2y + 1) = 0$

19. You are making a blanket with a fringe border of equal width on each side. *(Section 2.1 and Section 2.2)*

a. Write a polynomial that represents the perimeter of the blanket including the fringe.

b. Write a polynomial that represents the area of the blanket including the fringe.

c. Find the perimeter and the area of the blanket including the fringe when the width of the fringe is 4 inches.



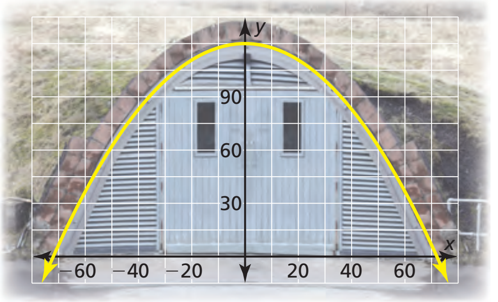
20. You are saving money to buy an electric guitar. You deposit \$1000 in an account that earns interest compounded annually. The expression $1000(1 + r)^2$ represents the balance after 2 years, where r is the annual interest rate in decimal form. *(Section 2.3)*

a. Write the polynomial in standard form that represents the balance of your account after 2 years.

b. The interest rate is 3%. What is the balance of your account after 2 years?

c. The guitar costs \$1100. Do you have enough money in your account *after 3 years*? Explain.

21. The front of a storage bunker can be modeled by $y = -\frac{5}{216}(x - 72)(x + 72)$, where x and y are measured in inches. The x-axis represents the ground. Find the width of the bunker at ground level. *(Section 2.4)*



2.5 Factoring $x^2 + bx + c$

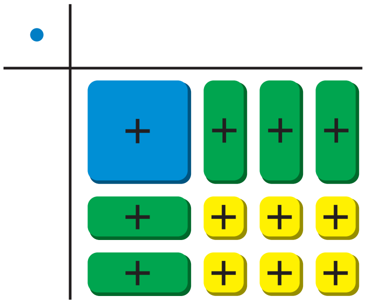
Essential Question How can you use algebra tiles to factor the trinomial $x^2 + bx + c$ into the product of two binomials?

EXPLORATION 1 Finding Binomial Factors

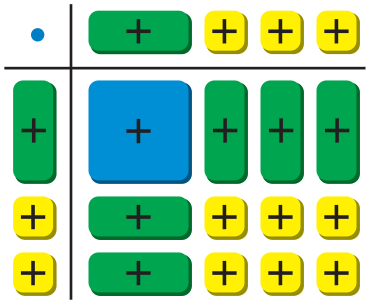
Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

Sample $x^2 + 5x + 6$

Step 1 Arrange algebra tiles that model $x^2 + 5x + 6$ into a rectangular array.



Step 2 Use additional algebra tiles to model the dimensions of the rectangle.

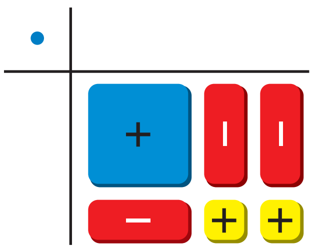


Step 3 Write the polynomial in factored form using the dimensions of the rectangle.

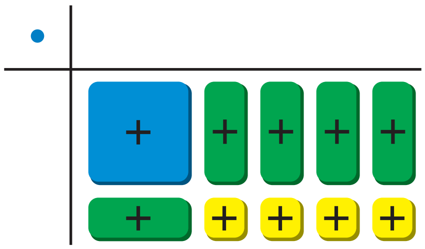
width length

$$\text{Area} = x^2 + 5x + 6 = (x + 2)(x + 3)$$

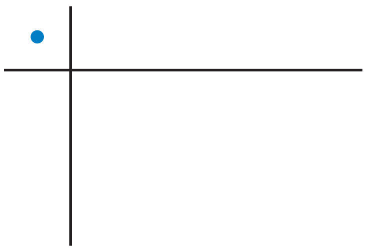
a. $x^2 - 3x + 2 =$ ______



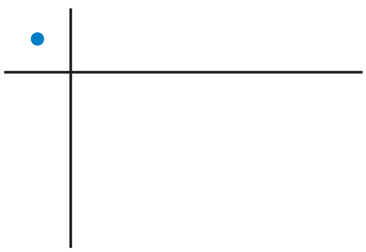
b. $x^2 + 5x + 4 =$ ______



c. $x^2 - 7x + 12 =$ ______



d. $x^2 + 7x + 12 =$ ______



REASONING ABSTRACTLY

To be proficient in math, you need to understand a situation abstractly and represent it symbolically.

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $x^2 + bx + c$ into the product of two binomials?

3. Describe a strategy for factoring the trinomial $x^2 + bx + c$ that does not use algebra tiles.

2.5 Lesson

Core Vocabulary

Previous
polynomial
FOIL Method
Zero-Product Property

What You Will Learn

- Factor $x^2 + bx + c$.
- Use factoring to solve real-life problems.

Factoring $x^2 + bx + c$

Writing a polynomial as a product of factors is called *factoring*. To factor $x^2 + bx + c$ as $(x + p)(x + q)$, you need to find p and q such that $p + q = b$ and $pq = c$.

$$\begin{aligned}(x + p)(x + q) &= x^2 + px + qx + pq \\ &= x^2 + (p + q)x + pq\end{aligned}$$

Core Concept

Factoring $x^2 + bx + c$ When c Is Positive

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is positive, p and q have the same sign as b.

Examples $x^2 + 6x + 5 = (x + 1)(x + 5)$

$x^2 - 6x + 5 = (x - 1)(x - 5)$

EXAMPLE 1 Factoring $x^2 + bx + c$ When b and c Are Positive

Factor $x^2 + 10x + 16$.

SOLUTION

Notice that $b = 10$ and $c = 16$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is also positive, p and q must each be positive so that $p + q$ is positive.

Find two positive integer factors of 16 whose sum is 10.

Factors of 16	Sum of factors
1, 16	17
2, 8	10
4, 4	8

The values of p and q are 2 and 8.

Check

Use the FOIL Method.

$(x + 2)(x + 8)$

$= x^2 + 8x + 2x + 16$

$= x^2 + 10x + 16$ ✓

▶ So, $x^2 + 10x + 16 = (x + 2)(x + 8)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

1. $x^2 + 7x + 6$
2. $x^2 + 9x + 8$

EXAMPLE 2 Factoring $x^2 + bx + c$ When b Is Negative and c Is Positive

Factor $x^2 - 8x + 12$.

SOLUTION

Notice that $b = -8$ and $c = 12$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is negative, p and q must each be negative so that $p + q$ is negative.

Find two negative integer factors of 12 whose sum is -8.

Factors of 12	$-1, -12$	$-2, -6$	$-3, -4$
Sum of factors	-13	-8	-7

The values of p and q are -2 and -6.

▶ So, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Check

Use the FOIL Method.

$(x - 2)(x - 6)$

$= x^2 - 6x - 2x + 12$

$= x^2 - 8x + 12$ ✓

Core Concept

Factoring $x^2 + bx + c$ When c Is Negative

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is negative, p and q have different signs.

Example $x^2 - 4x - 5 = (x + 1)(x - 5)$

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor $x^2 + 4x - 21$.

SOLUTION

Notice that $b = 4$ and $c = -21$. Because c is negative, the factors p and q must have different signs so that pq is negative.

Find two integer factors of -21 whose sum is 4.

Factors of -21	$-21, 1$	$-1, 21$	$-7, 3$	$-3, 7$
Sum of factors	-20	20	-4	4

The values of p and q are -3 and 7.

▶ So, $x^2 + 4x - 21 = (x - 3)(x + 7)$.

Check

Use the FOIL Method.

$(x - 3)(x + 7)$

$= x^2 + 7x - 3x - 21$

$= x^2 + 4x - 21$ ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

3. $w^2 - 4w + 3$

4. $n^2 - 12n + 35$

5. $x^2 - 14x + 24$

6. $x^2 + 2x - 15$

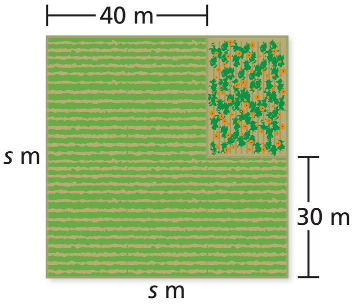
7. $y^2 + 13y - 30$

8. $v^2 - v - 42$

Solving Real-Life Problems

EXAMPLE 4 **Solving a Real-Life Problem**

A farmer plants a rectangular pumpkin patch in the northeast corner of a square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?



SOLUTION

1. **Understand the Problem** You are given the area of the pumpkin patch, the difference of the side length of the square plot and the length of the pumpkin patch, and the difference of the side length of the square plot and the width of the pumpkin patch.

2. **Make a Plan** The length of the pumpkin patch is $(s - 30)$ meters and the width is $(s - 40)$ meters. Write and solve an equation to find the side length s. Then use the solution to find the area of the square plot of land.

3. **Solve the Problem** Use the equation for the area of a rectangle to write and solve an equation to find the side length s of the square plot of land.

$$600 = (s - 30)(s - 40) \quad \text{Write an equation.}$$
$$600 = s^2 - 70s + 1200 \quad \text{Multiply.}$$
$$0 = s^2 - 70s + 600 \quad \text{Subtract 600 from each side.}$$
$$0 = (s - 10)(s - 60) \quad \text{Factor the polynomial.}$$
$$s - 10 = 0 \quad or \quad s - 60 = 0 \quad \text{Zero-Product Property}$$
$$s = 10 \quad or \quad s = 60 \quad \text{Solve for } s.$$

STUDY TIP

The diagram shows that the side length is more than 40 meters, so a side length of 10 meters does not make sense in this situation. The side length is 60 meters.

▶ So, the area of the square plot of land is $60(60) = 3600$ square meters.

4. **Look Back** Use the diagram to check that you found the correct side length. Using $s = 60$, the length of the pumpkin patch is $60 - 30 = 30$ meters and the width is $60 - 40 = 20$ meters. So, the area of the pumpkin patch is 600 square meters. This matches the given information and confirms the side length is 60 meters, which gives an area of 3600 square meters.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

9. **WHAT IF?** The area of the pumpkin patch is 200 square meters. What is the area of the square plot of land?

Concept Summary

Factoring $x^2 + bx + c$ as $(x + p)(x + q)$

The diagram shows the relationships between the signs of b and c and the signs of p and q.

$x^2 + bx + c = (x + p)(x + q)$

- c is positive. b is positive. → p and q are positive.
- c is positive. b is negative. → p and q are negative.
- c is negative. → p and q have different signs.

2.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** You are factoring $x^2 + 11x - 26$. What do the signs of the terms tell you about the factors? Explain.

2. **OPEN-ENDED** Write a trinomial that can be factored as $(x + p)(x + q)$, where p and q are positive.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $x^2 + 8x + 7$
4. $z^2 + 10z + 21$
5. $n^2 + 9n + 20$
6. $s^2 + 11s + 30$
7. $h^2 + 11h + 18$
8. $y^2 + 13y + 40$

In Exercises 9–14, factor the polynomial. *(See Example 2.)*

9. $v^2 - 5v + 4$
10. $x^2 - 13x + 22$
11. $d^2 - 5d + 6$
12. $k^2 - 10k + 24$
13. $w^2 - 17w + 72$
14. $j^2 - 13j + 42$

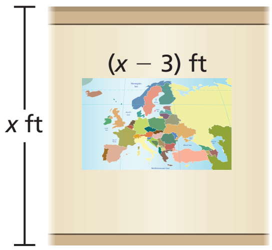
In Exercises 15–24, factor the polynomial. *(See Example 3.)*

15. $x^2 + 3x - 4$
16. $z^2 + 7z - 18$
17. $n^2 + 4n - 12$
18. $s^2 + 3s - 40$
19. $y^2 + 2y - 48$
20. $h^2 + 6h - 27$
21. $x^2 - x - 20$
22. $m^2 - 6m - 7$
23. $-6t - 16 + t^2$
24. $-7y + y^2 - 30$

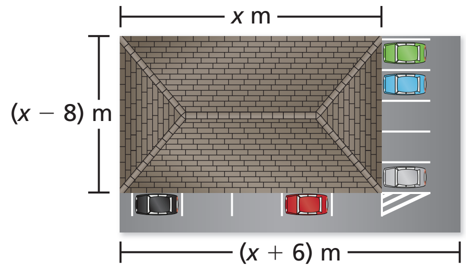
25. **MODELING WITH MATHEMATICS** A projector displays an image on a wall. The area (in square feet) of the projection is represented by $x^2 - 8x + 15$.

    a. Write a binomial that represents the height of the projection.

    b. Find the perimeter of the projection when the height of the wall is 8 feet.



26. **MODELING WITH MATHEMATICS** A dentist's office and parking lot are on a rectangular piece of land. The area (in square meters) of the land is represented by $x^2 + x - 30$.



    a. Write a binomial that represents the width of the land.

    b. Find the area of the land when the length of the dentist's office is 20 meters.

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in factoring the polynomial.

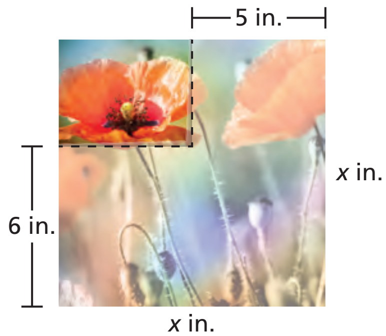
27. $x^2 + 14x + 48 = (x + 4)(x + 12)$

28. $s^2 - 17s - 60 = (s - 5)(s - 12)$

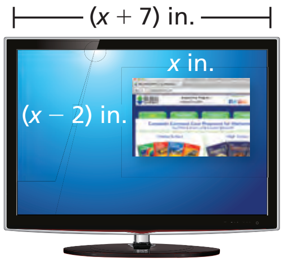
In Exercises 29–38, solve the equation.

29. $m^2 + 3m + 2 = 0$
30. $n^2 - 9n + 18 = 0$
31. $x^2 + 5x - 14 = 0$
32. $v^2 + 11v - 26 = 0$
33. $t^2 + 15t = -36$
34. $n^2 - 5n = 24$
35. $a^2 + 5a - 20 = 30$
36. $y^2 - 2y - 8 = 7$
37. $m^2 + 10 = 15m - 34$
38. $b^2 + 5 = 8b - 10$

39. **MODELING WITH MATHEMATICS** You trimmed a large square picture so that you could fit it into a frame. The area of the cut picture is 20 square inches. What is the area of the original picture? *(See Example 4.)*



40. **MODELING WITH MATHEMATICS** A web browser is open on your computer screen.

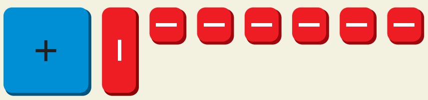


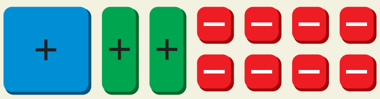
a. The area of the browser window is 24 square inches. Find the length of the browser window x.

b. The browser covers $\frac{3}{13}$ of the screen. What are the dimensions of the screen?

41. **MAKING AN ARGUMENT** Your friend says there are six integer values of b for which the trinomial $x^2 + bx - 12$ has two binomial factors of the form $(x + p)$ and $(x + q)$. Is your friend correct? Explain.

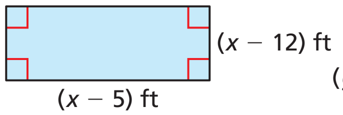
42. **THOUGHT PROVOKING** Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a. 

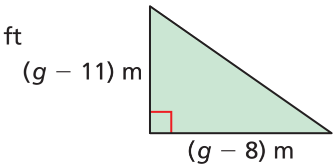
b. 

MATHEMATICAL CONNECTIONS **In Exercises 43 and 44, find the dimensions of the polygon with the given area.**

43. Area $= 44$ ft^2

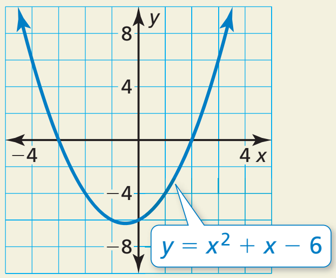


44. Area $= 35$ m^2



45. **REASONING** Write an equation of the form $x^2 + bx + c = 0$ that has the solutions $x = -4$ and $x = 6$. Explain how you found your answer.

46. **HOW DO YOU SEE IT?** The graph of $y = x^2 + x - 6$ is shown.



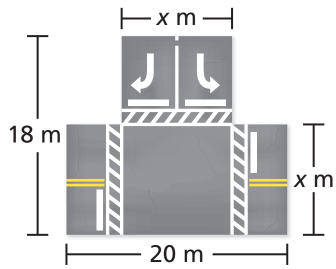
a. Explain how you can use the graph to factor the polynomial $x^2 + x - 6$.

b. Factor the polynomial.

47. **PROBLEM SOLVING** Road construction workers are paving the area shown.

a. Write an expression that represents the area being paved.

b. The area being paved is 280 square meters. Write and solve an equation to find the width of the road x.



USING STRUCTURE **In Exercises 48–51, factor the polynomial.**

48. $x^2 + 6xy + 8y^2$

49. $r^2 + 7rs + 12s^2$

50. $a^2 + 11ab - 26b^2$

51. $x^2 - 2xy - 35y^2$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Skills Review Handbook)*

52. $p - 9 = 0$

53. $z + 12 = -5$

54. $6 = \frac{c}{-7}$

55. $4k = 0$

2.6 Factoring $ax^2 + bx + c$

Essential Question

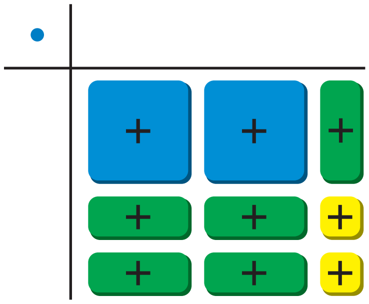
How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

EXPLORATION 1 Finding Binomial Factors

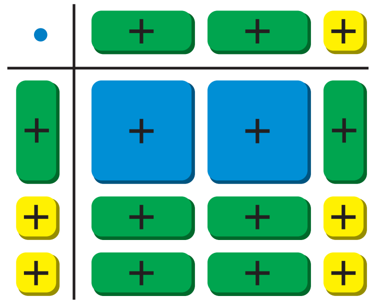
Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

Sample $2x^2 + 5x + 2$

Step 1 Arrange algebra tiles that model $2x^2 + 5x + 2$ into a rectangular array.



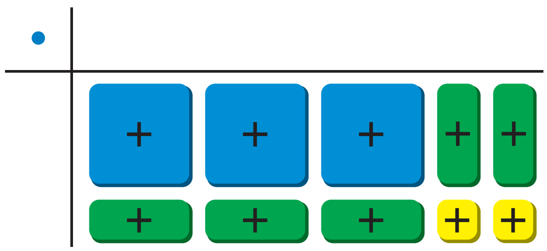
Step 2 Use additional algebra tiles to model the dimensions of the rectangle.



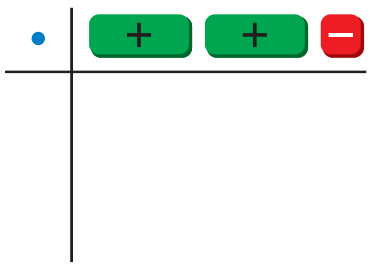
Step 3 Write the polynomial in factored form using the dimensions of the rectangle.

$$\text{Area} = 2x^2 + 5x + 2 = \overbrace{(x + 2)}^{\text{width}}\overbrace{(2x + 1)}^{\text{length}}$$

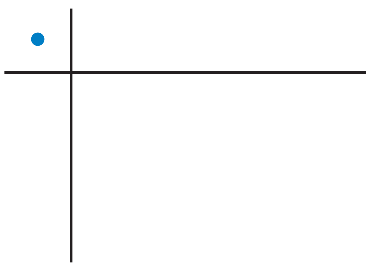
a. $3x^2 + 5x + 2 =$ ______



b. $4x^2 + 4x - 3 =$ ______



c. $2x^2 - 11x + 5 =$ ______



USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider the available tools, including concrete models, when solving a mathematical problem.

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?
3. Is it possible to factor the trinomial $2x^2 + 2x + 1$? Explain your reasoning.

2.6 Lesson

Core Vocabulary

Previous
polynomial
greatest common factor (GCF)
Zero-Product Property

What You Will Learn

- Factor $ax^2 + bx + c$.
- Use factoring to solve real-life problems.

Factoring $ax^2 + bx + c$

In Section 2.5, you factored polynomials of the form $ax^2 + bx + c$, where $a = 1$. To factor polynomials of the form $ax^2 + bx + c$, where $a \neq 1$, first look for the GCF of the terms of the polynomial and then factor further if possible.

EXAMPLE 1 Factoring Out the GCF

Factor $5x^2 + 15x + 10$.

SOLUTION

Notice that the GCF of the terms $5x^2$, $15x$, and 10 is 5.

$5x^2 + 15x + 10 = 5(x^2 + 3x + 2)$ Factor out GCF.

$= 5(x + 1)(x + 2)$ Factor $x^2 + 3x + 2$.

▶ So, $5x^2 + 15x + 10 = 5(x + 1)(x + 2)$.

When there is no GCF, consider the possible factors of a and c.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When ac Is Positive

Factor each polynomial.

a. $4x^2 + 13x + 3$ **b.** $3x^2 - 7x + 2$

SOLUTION

a. There is no GCF, so you need to consider the possible factors of a and c. Because b and c are both positive, the factors of c must be positive. Use a table to organize information about the factors of a and c.

STUDY TIP

You must consider the order of the factors of 3, because the middle terms formed by the possible factorizations are different.

Factors of 4	Factors of 3	Possible factorization	Middle term	
1, 4	1, 3	$(x + 1)(4x + 3)$	$3x + 4x = 7x$	✗
1, 4	3, 1	$(x + 3)(4x + 1)$	$x + 12x = 13x$	✓
2, 2	1, 3	$(2x + 1)(2x + 3)$	$6x + 2x = 8x$	✗

▶ So, $4x^2 + 13x + 3 = (x + 3)(4x + 1)$.

b. There is no GCF, so you need to consider the possible factors of a and c. Because b is negative and c is positive, both factors of c must be negative. Use a table to organize information about the factors of a and c.

Factors of 3	Factors of 2	Possible factorization	Middle term	
1, 3	−1, −2	$(x - 1)(3x - 2)$	$-2x - 3x = -5x$	✗
1, 3	−2, −1	$(x - 2)(3x - 1)$	$-x - 6x = -7x$	✓

▶ So, $3x^2 - 7x + 2 = (x - 2)(3x - 1)$.

EXAMPLE 3 Factoring $ax^2 + bx + c$ When ac Is Negative

Factor $2x^2 - 5x - 7$.

SOLUTION

There is no GCF, so you need to consider the possible factors of a and c. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 2	Factors of −7	Possible factorization	Middle term	
1, 2	1, −7	$(x + 1)(2x - 7)$	$-7x + 2x = -5x$	✓
1, 2	7, −1	$(x + 7)(2x - 1)$	$-x + 14x = 13x$	✗
1, 2	−1, 7	$(x - 1)(2x + 7)$	$7x - 2x = 5x$	✗
1, 2	−7, 1	$(x - 7)(2x + 1)$	$x - 14x = -13x$	✗

► So, $2x^2 - 5x - 7 = (x + 1)(2x - 7)$.

STUDY TIP

When a is negative, factor -1 from each term of $ax^2 + bx + c$. Then factor the resulting trinomial as in the previous examples.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-4x^2 - 8x + 5$.

SOLUTION

Step 1 Factor -1 from each term of the trinomial.

$$-4x^2 - 8x + 5 = -(4x^2 + 8x - 5)$$

Step 2 Factor the trinomial $4x^2 + 8x - 5$. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 4	Factors of −5	Possible factorization	Middle term	
1, 4	1, −5	$(x + 1)(4x - 5)$	$-5x + 4x = -x$	✗
1, 4	5, −1	$(x + 5)(4x - 1)$	$-x + 20x = 19x$	✗
1, 4	−1, 5	$(x - 1)(4x + 5)$	$5x - 4x = x$	✗
1, 4	−5, 1	$(x - 5)(4x + 1)$	$x - 20x = -19x$	✗
2, 2	1, −5	$(2x + 1)(2x - 5)$	$-10x + 2x = -8x$	✗
2, 2	−1, 5	$(2x - 1)(2x + 5)$	$10x - 2x = 8x$	✓

► So, $-4x^2 - 8x + 5 = -(2x - 1)(2x + 5)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

1. $8x^2 - 56x + 48$ **2.** $14x^2 + 31x + 15$ **3.** $2x^2 - 7x + 5$

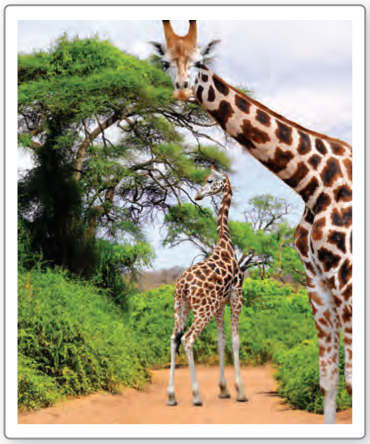
4. $3x^2 - 14x + 8$ **5.** $4x^2 - 19x - 5$ **6.** $6x^2 + x - 12$

7. $-2y^2 - 5y - 3$ **8.** $-5m^2 + 6m - 1$ **9.** $-3x^2 - x + 2$

Solving Real-Life Problems

EXAMPLE 5 Solving a Real-Life Problem

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. What is the width of the reserve?



SOLUTION

Use the formula for the area of a rectangle to write an equation for the area of the reserve. Let w represent the width. Then $2w + 1$ represents the length. Solve for w.

$w(2w + 1) = 55$ Area of the reserve

$2w^2 + w = 55$ Distributive Property

$2w^2 + w - 55 = 0$ Subtract 55 from each side.

Factor the left side of the equation. There is no GCF, so you need to consider the possible factors of a and c. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 2	Factors of −55	Possible factorization	Middle term	
1, 2	$1, -55$	$(w + 1)(2w - 55)$	$-55w + 2w = -53w$	✗
1, 2	$55, -1$	$(w + 55)(2w - 1)$	$-w + 110w = 109w$	✗
1, 2	$-1, 55$	$(w - 1)(2w + 55)$	$55w - 2w = 53w$	✗
1, 2	$-55, 1$	$(w - 55)(2w + 1)$	$w - 110w = -109w$	✗
1, 2	$5, -11$	$(w + 5)(2w - 11)$	$-11w + 10w = -w$	✗
1, 2	$11, -5$	$(w + 11)(2w - 5)$	$-5w + 22w = 17w$	✗
1, 2	$-5, 11$	$(w - 5)(2w + 11)$	$11w - 10w = w$	✓
1, 2	$-11, 5$	$(w - 11)(2w + 5)$	$5w - 22w = -17w$	✗

So, you can rewrite $2w^2 + w - 55$ as $(w - 5)(2w + 11)$. Write the equation with the left side factored and continue solving for w.

$(w - 5)(2w + 11) = 0$ Rewrite equation with left side factored.

$w - 5 = 0$ *or* $2w + 11 = 0$ Zero-Product Property

$w = 5$ *or* $w = -\frac{11}{2}$ Solve for w.

A negative width does not make sense, so you should use the positive solution.

▶ So, the width of the reserve is 5 miles.

Check

Use mental math.

The width is 5 miles, so the length is $5(2) + 1 = 11$ miles and the area is $5(11) = 55$ square miles. ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

10. WHAT IF? The area of the reserve is 136 square miles. How wide is the reserve?

2.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** What is the greatest common factor of the terms of $3y^2 - 21y + 36$?

2. **WRITING** Compare factoring $6x^2 - x - 2$ with factoring $x^2 - x - 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $3x^2 + 3x - 6$
4. $8v^2 + 8v - 48$
5. $4k^2 + 28k + 48$
6. $6y^2 - 24y + 18$
7. $7b^2 - 63b + 140$
8. $9r^2 - 36r - 45$

In Exercises 9–16, factor the polynomial. *(See Examples 2 and 3.)*

9. $3h^2 + 11h + 6$
10. $8m^2 + 30m + 7$
11. $6x^2 - 5x + 1$
12. $10w^2 - 31w + 15$
13. $3n^2 + 5n - 2$
14. $4z^2 + 4z - 3$
15. $8g^2 - 10g - 12$
16. $18v^2 - 15v - 18$

In Exercises 17–22, factor the polynomial. *(See Example 4.)*

17. $-3t^2 + 11t - 6$
18. $-7v^2 - 25v - 12$
19. $-4c^2 + 19c + 5$
20. $-8h^2 - 13h + 6$
21. $-15w^2 - w + 28$
22. $-22d^2 + 29d - 9$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in factoring the polynomial.**

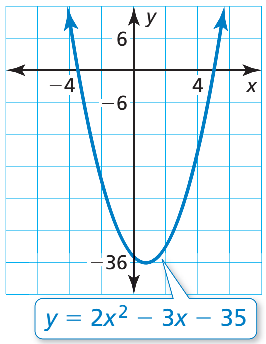
23. ✗
$$2x^2 - 2x - 24 = 2(x^2 - 2x - 24)$$
$$= 2(x - 6)(x + 4)$$

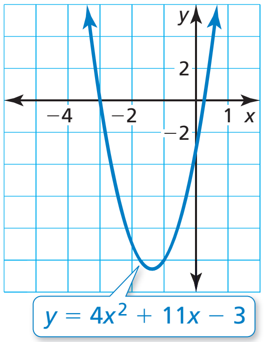
24. ✗
$$6x^2 - 7x - 3 = (3x - 3)(2x + 1)$$

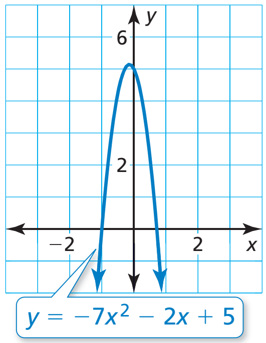
In Exercises 25–28, solve the equation.

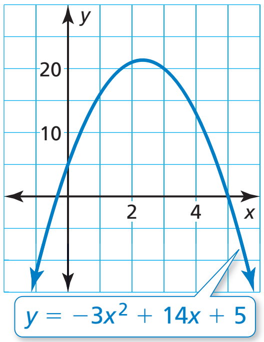
25. $5x^2 - 5x - 30 = 0$
26. $2k^2 - 5k - 18 = 0$
27. $-12n^2 - 11n = -15$
28. $14b^2 - 2 = -3b$

In Exercises 29–32, find the x-coordinates of the points where the graph crosses the x-axis.

29.


30.


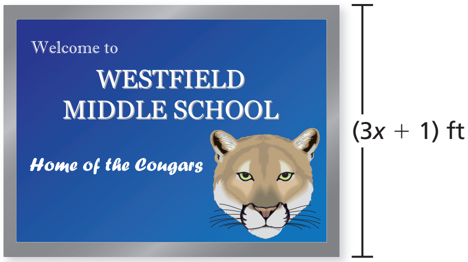
31.


32.


33. **MODELING WITH MATHEMATICS** The area (in square feet) of the school sign can be represented by $15x^2 - x - 2$.

a. Write an expression that represents the length of the sign.

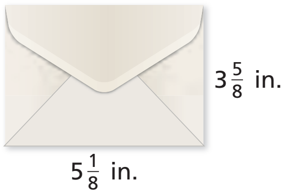
b. Describe two ways to find the area of the sign when $x = 3$.



34. **MODELING WITH MATHEMATICS** The height h (in feet) above the water of a cliff diver is modeled by $h = -16t^2 + 8t + 80$, where t is the time (in seconds). What does the constant term represent? How long is the diver in the air?

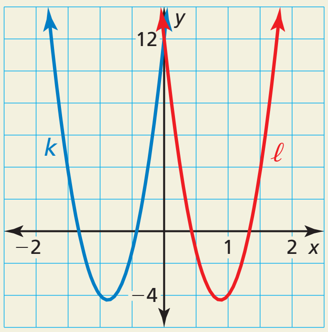
35. **MODELING WITH MATHEMATICS** The Parthenon in Athens, Greece, is an ancient structure that has a rectangular base. The length of the base of the Parthenon is 8 meters more than twice its width. The area of the base is about 2170 square meters. Find the length and width of the base. *(See Example 5.)*

36. **MODELING WITH MATHEMATICS** The length of a rectangular birthday party invitation is 1 inch less than twice its width. The area of the invitation is 15 square inches. Will the invitation fit in the envelope shown without being folded? Explain.



37. **OPEN-ENDED** Write a binomial whose terms have a GCF of $3x$.

38. **HOW DO YOU SEE IT?** Without factoring, determine which of the graphs represents the function $g(x) = 21x^2 + 37x + 12$ and which represents the function $h(x) = 21x^2 - 37x + 12$. Explain your reasoning.



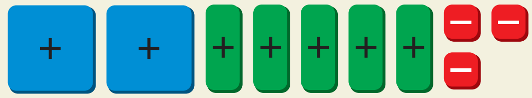
39. **REASONING** When is it not possible to factor $ax^2 + bx + c$, where $a \neq 1$? Give an example.

40. **MAKING AN ARGUMENT** Your friend says that to solve the equation $5x^2 + x - 4 = 2$, you should start by factoring the left side as $(5x - 4)(x + 1)$. Is your friend correct? Explain.

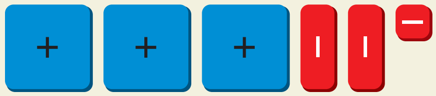
41. **REASONING** For what values of t can $2x^2 + tx + 10$ be written as the product of two binomials?

42. **THOUGHT PROVOKING** Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a.

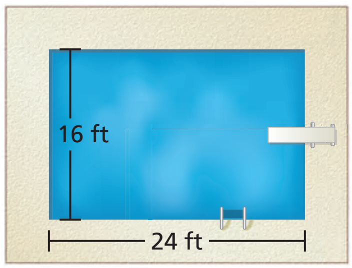


b.



43. **MATHEMATICAL CONNECTIONS** The length of a rectangle is 1 inch more than twice its width. The value of the area of the rectangle (in square inches) is 5 more than the value of the perimeter of the rectangle (in inches). Find the width.

44. **PROBLEM SOLVING** A rectangular swimming pool is bordered by a concrete patio. The width of the patio is the same on every side. The area of the surface of the pool is equal to the area of the patio. What is the width of the patio?



In Exercises 45–48, factor the polynomial.

45. $4k^2 + 7jk - 2j^2$

46. $6x^2 + 5xy - 4y^2$

47. $-6a^2 + 19ab - 14b^2$

48. $18m^3 + 39m^2n - 15mn^2$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. *(Section 1.1)*

49. $h(x) = 3|x|$

50. $v(x) = |x - 5|$

51. $g(x) = |x| + 1$

52. $r(x) = -2|x|$

Evaluate the expression. *(Section 1.5)*

53. $-\sqrt[3]{216}$

54. $\sqrt[5]{-32}$

55. $16^{5/4}$

56. $(-27)^{2/3}$

2.7 Factoring Special Products

Essential Question How can you recognize and factor special products?

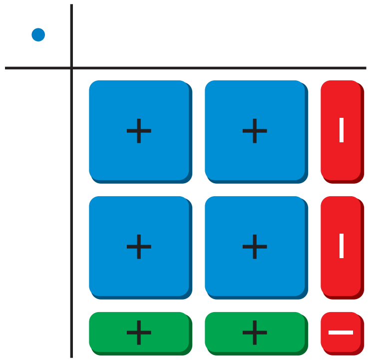
EXPLORATION 1 Factoring Special Products

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a "special product" that you studied in Section 2.3.

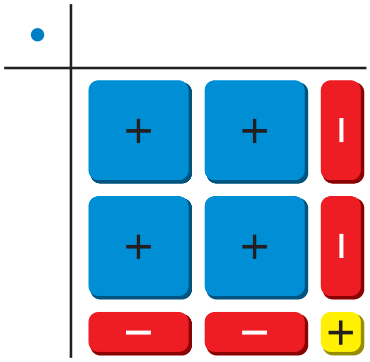
LOOKING FOR STRUCTURE

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

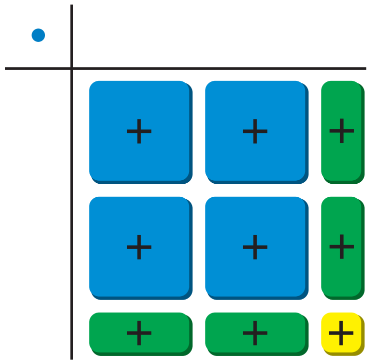
a. $4x^2 - 1 =$ ______



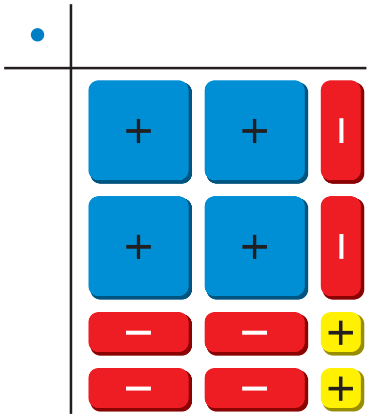
b. $4x^2 - 4x + 1 =$ ______



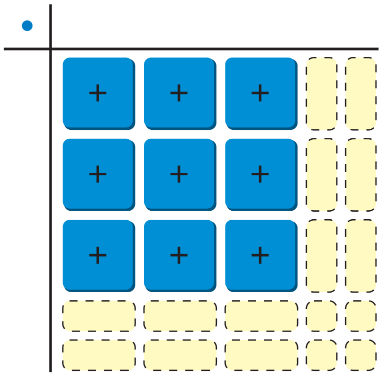
c. $4x^2 + 4x + 1 =$ ______



d. $4x^2 - 6x + 2 =$ ______



EXPLORATION 2 Factoring Special Products



Work with a partner. Use algebra tiles to complete the rectangular array at the left in three different ways, so that each way represents a different special product. Write each special product in standard form and in factored form.

Communicate Your Answer

3. How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.

4. Use the strategy you described in Question 3 to factor each polynomial.

   a. $25x^2 + 10x + 1$ **b.** $25x^2 - 10x + 1$ **c.** $25x^2 - 1$

2.7 Lesson

Core Vocabulary

Previous
polynomial
trinomial

What You Will Learn

- Factor the difference of two squares.
- Factor perfect square trinomials.
- Use factoring to solve real-life problems.

Factoring the Difference of Two Squares

You can use special product patterns to factor polynomials.

Core Concept

Difference of Two Squares Pattern

Algebra	**Example**
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

EXAMPLE 1 Factoring the Difference of Two Squares

Factor (a) $x^2 - 25$ and (b) $4z^2 - 1$.

SOLUTION

a. $x^2 - 25 = x^2 - 5^2$ Write as $a^2 - b^2$.

$= (x + 5)(x - 5)$ Difference of two squares pattern

▶ So, $x^2 - 25 = (x + 5)(x - 5)$.

b. $4z^2 - 1 = (2z)^2 - 1^2$ Write as $a^2 - b^2$.

$= (2z + 1)(2z - 1)$ Difference of two squares pattern

▶ So, $4z^2 - 1 = (2z + 1)(2z - 1)$.

EXAMPLE 2 Evaluating a Numerical Expression

Use a special product pattern to evaluate the expression $54^2 - 48^2$.

SOLUTION

Notice that $54^2 - 48^2$ is a difference of two squares. So, you can rewrite the expression in a form that it is easier to evaluate using the difference of two squares pattern.

$54^2 - 48^2 = (54 + 48)(54 - 48)$ Difference of two squares pattern

$= 102(6)$ Simplify.

$= 612$ Multiply.

▶ So, $54^2 - 48^2 = 612$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

1. $x^2 - 36$ **2.** $100 - m^2$ **3.** $9n^2 - 16$ **4.** $16h^2 - 49$

Use a special product pattern to evaluate the expression.

5. $36^2 - 34^2$ **6.** $47^2 - 44^2$ **7.** $55^2 - 50^2$ **8.** $28^2 - 24^2$

Factoring Perfect Square Trinomials

Core Concept

Perfect Square Trinomial Pattern

Algebra	**Example**
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$ $= (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$ $= (x - 3)^2$

EXAMPLE 3 Factoring Perfect Square Trinomials

Factor each polynomial.

a. $n^2 + 8n + 16$ **b.** $4x^2 - 12x + 9$

SOLUTION

a. $n^2 + 8n + 16 = n^2 + 2(n)(4) + 4^2$ Write as $a^2 + 2ab + b^2$.

$= (n + 4)^2$ Perfect square trinomial pattern

▶ So, $n^2 + 8n + 16 = (n + 4)^2$.

b. $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ Write as $a^2 - 2ab + b^2$.

$= (2x - 3)^2$ Perfect square trinomial pattern

▶ So, $4x^2 - 12x + 9 = (2x - 3)^2$.

EXAMPLE 4 Solving a Polynomial Equation

Solve $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$.

SOLUTION

$x^2 + \frac{2}{3}x + \frac{1}{9} = 0$ Write equation.

$9x^2 + 6x + 1 = 0$ Multiply each side by 9.

$(3x)^2 + 2(3x)(1) + 1^2 = 0$ Write left side as $a^2 + 2ab + b^2$.

$(3x + 1)^2 = 0$ Perfect square trinomial pattern

$3x + 1 = 0$ Zero-Product Property

$x = -\frac{1}{3}$ Solve for x.

▶ The solution is $x = -\frac{1}{3}$.

LOOKING FOR STRUCTURE

Equations of the form $(x + a)^2 = 0$ always have repeated roots of $x = -a$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial.

9. $m^2 - 2m + 1$ **10.** $d^2 - 10d + 25$ **11.** $9z^2 + 36z + 36$

Solve the equation.

12. $a^2 + 6a + 9 = 0$ **13.** $w^2 - \frac{7}{3}w + \frac{49}{36} = 0$ **14.** $n^2 - 81 = 0$

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**

A bird picks up a golf ball and drops it while flying. The function represents the height y (in feet) of the golf ball t seconds after it is dropped. The ball hits the top of a 32-foot-tall pine tree. After how many seconds does the ball hit the tree?

SOLUTION

1. **Understand the Problem** You are given the height of the golf ball as a function of the amount of time after it is dropped and the height of the tree that the golf ball hits. You are asked to determine how many seconds it takes for the ball to hit the tree.

2. **Make a Plan** Use the function for the height of the golf ball. Substitute the height of the tree for y and solve for the time t.

3. **Solve the Problem** Substitute 32 for y and solve for t.

$$y = 81 - 16t^2 \quad \text{Write equation.}$$

$$32 = 81 - 16t^2 \quad \text{Substitute 32 for } y.$$

$$0 = 49 - 16t^2 \quad \text{Subtract 32 from each side.}$$

$$0 = 7^2 - (4t)^2 \quad \text{Write as } a^2 - b^2.$$

$$0 = (7 + 4t)(7 - 4t) \quad \text{Difference of two squares pattern}$$

$$7 + 4t = 0 \quad or \quad 7 - 4t = 0 \quad \text{Zero-Product Property}$$

$$t = -\frac{7}{4} \quad or \quad t = \frac{7}{4} \quad \text{Solve for } t.$$

A negative time does not make sense in this situation.

▶ So, the golf ball hits the tree after $\frac{7}{4}$, or 1.75 seconds.

4. **Look Back** Check your solution, as shown, by substituting $t = \frac{7}{4}$ into the equation $32 = 81 - 16t^2$. Then verify that a time of $\frac{7}{4}$ seconds gives a height of 32 feet.

Check

$$32 = 81 - 16t^2$$

$$32 \stackrel{?}{=} 81 - 16\left(\frac{7}{4}\right)^2$$

$$32 \stackrel{?}{=} 81 - 16\left(\frac{49}{16}\right)$$

$$32 \stackrel{?}{=} 81 - 49$$

$$32 = 32 \ \checkmark$$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

15. **WHAT IF?** The golf ball does not hit the pine tree. After how many seconds does the ball hit the ground?

2.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you use the perfect square trinomial pattern to factor $y^2 + 16y + 64$? Explain.

2. **WHICH ONE DOESN'T BELONG?** Which polynomial does *not* belong with the other three? Explain your reasoning.

$n^2 - 4$	$g^2 - 6g + 9$	$r^2 + 12r + 36$	$k^2 + 25$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $m^2 - 49$
4. $z^2 - 81$
5. $64 - 81d^2$
6. $25 - 4x^2$
7. $225a^2 - 36b^2$
8. $16x^2 - 169y^2$

In Exercises 9–14, use a special product pattern to evaluate the expression. *(See Example 2.)*

9. $12^2 - 9^2$
10. $19^2 - 11^2$
11. $78^2 - 72^2$
12. $54^2 - 52^2$
13. $53^2 - 47^2$
14. $39^2 - 36^2$

In Exercises 15–22, factor the polynomial. *(See Example 3.)*

15. $h^2 + 12h + 36$
16. $p^2 + 30p + 225$
17. $y^2 - 22y + 121$
18. $x^2 - 4x + 4$
19. $a^2 - 28a + 196$
20. $m^2 + 24m + 144$
21. $25n^2 + 20n + 4$
22. $49a^2 - 14a + 1$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in factoring the polynomial.**

23. ✗
$$n^2 - 64 = n^2 - 8^2$$
$$= (n - 8)^2$$

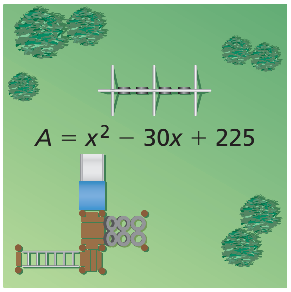
24. ✗
$$y^2 - 6y + 9 = y^2 - 2(y)(3) + 3^2$$
$$= (y - 3)(y + 3)$$

25. **MODELING WITH MATHEMATICS** The area (in square centimeters) of a square coaster can be represented by $d^2 + 8d + 16$.

   a. Write an expression that represents the side length of the coaster.

   b. Write an expression for the perimeter of the coaster.

26. **MODELING WITH MATHEMATICS** The polynomial represents the area (in square feet) of the square playground.



   a. Write a polynomial that represents the side length of the playground.

   b. Write an expression for the perimeter of the playground.

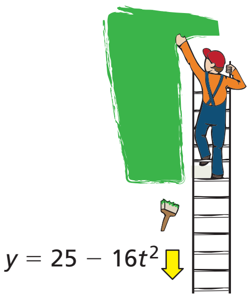
In Exercises 27–34, solve the equation. *(See Example 4.)*

27. $z^2 - 4 = 0$
28. $4x^2 = 49$
29. $k^2 - 16k + 64 = 0$
30. $s^2 + 20s + 100 = 0$
31. $n^2 + 9 = 6n$
32. $y^2 = 12y - 36$
33. $y^2 + \frac{1}{2}y = -\frac{1}{16}$
34. $-\frac{4}{3}x + \frac{4}{9} = -x^2$

In Exercises 35–40, factor the polynomial.

35. $3z^2 - 27$
36. $2m^2 - 50$
37. $4y^2 - 16y + 16$
38. $8k^2 + 80k + 200$
39. $50y^2 + 120y + 72$
40. $27m^2 - 36m + 12$

41. **MODELING WITH MATHEMATICS** While standing on a ladder, you drop a paintbrush. The function represents the height y (in feet) of the paintbrush t seconds after it is dropped. After how many seconds does the paintbrush land on the ground? *(See Example 5.)*



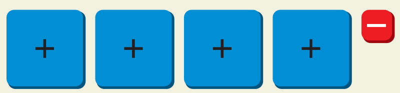
42. **MODELING WITH MATHEMATICS** The function represents the height y (in feet) of a grasshopper jumping straight up from the ground t seconds after the start of the jump. After how many seconds is the grasshopper 1 foot off the ground?

43. **REASONING** Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial is a perfect square trinomial.

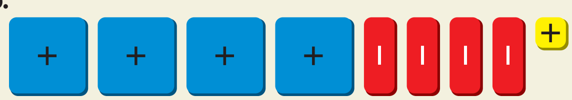
a. $w^2 + 18w + 84$ **b.** $y^2 - 10y + 23$

44. **THOUGHT PROVOKING** Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a.

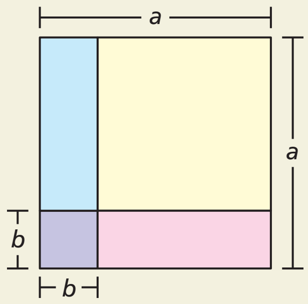


b.



45. **COMPARING METHODS** Describe two methods you can use to simplify $(2x - 5)^2 - (x - 4)^2$. Which one would you use? Explain.

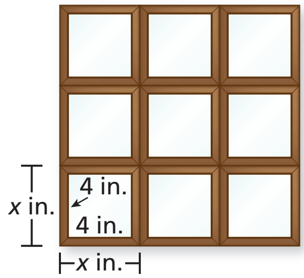
46. **HOW DO YOU SEE IT?** The figure shows a large square with an area of a^2 that contains a smaller square with an area of b^2.



a. Describe the regions that represent $a^2 - b^2$. How can you rearrange these regions to show that $a^2 - b^2 = (a + b)(a - b)$?

b. How can you use the figure to show that $(a - b)^2 = a^2 - 2ab + b^2$?

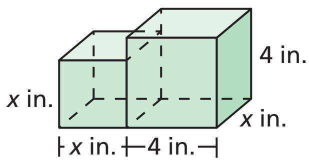
47. **PROBLEM SOLVING** You hang nine identical square picture frames on a wall.



a. Write a polynomial that represents the area of the picture frames, not including the pictures.

b. The area in part (a) is 81 square inches. What is the side length of one of the picture frames? Explain your reasoning.

48. **MATHEMATICAL CONNECTIONS** The composite solid is made up of a cube and a rectangular prism.



a. Write a polynomial that represents the volume of the composite solid.

b. The volume of the composite solid is equal to $25x$. What is the value of x? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the prime factorization of the number. *(Skills Review Handbook)*

49. 50 **50.** 44 **51.** 85 **52.** 96

Find the inverse of the function. Then graph the function and its inverse. *(Section 1.3)*

53. $f(x) = x - 3$ **54.** $f(x) = -2x + 5$ **55.** $f(x) = \frac{1}{2}x - 1$

2.8 Factoring Polynomials Completely

Essential Question

How can you factor a polynomial completely?

EXPLORATION 1 Writing a Product of Linear Factors

Work with a partner. Write the product represented by the algebra tiles. Then multiply to write the polynomial in standard form.

a. 

b. 

c. 

d. 

e. 

f. 

REASONING ABSTRACTLY

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

EXPLORATION 2 Matching Standard and Factored Forms

Work with a partner. Match the standard form of the polynomial with the equivalent factored form. Explain your strategy.

a. $x^3 + x^2$	**A.** $x(x + 1)(x - 1)$
b. $x^3 - x$	**B.** $x(x - 1)^2$
c. $x^3 + x^2 - 2x$	**C.** $x(x + 1)^2$
d. $x^3 - 4x^2 + 4x$	**D.** $x(x + 2)(x - 1)$
e. $x^3 - 2x^2 - 3x$	**E.** $x(x - 1)(x - 2)$
f. $x^3 - 2x^2 + x$	**F.** $x(x + 2)(x - 2)$
g. $x^3 - 4x$	**G.** $x(x - 2)^2$
h. $x^3 + 2x^2$	**H.** $x(x + 2)^2$
i. $x^3 - x^2$	**I.** $x^2(x - 1)$
j. $x^3 - 3x^2 + 2x$	**J.** $x^2(x + 1)$
k. $x^3 + 2x^2 - 3x$	**K.** $x^2(x - 2)$
l. $x^3 - 4x^2 + 3x$	**L.** $x^2(x + 2)$
m. $x^3 - 2x^2$	**M.** $x(x + 3)(x - 1)$
n. $x^3 + 4x^2 + 4x$	**N.** $x(x + 1)(x - 3)$
o. $x^3 + 2x^2 + x$	**O.** $x(x - 1)(x - 3)$

Communicate Your Answer

3. How can you factor a polynomial completely?

4. Use your answer to Question 3 to factor each polynomial completely.

a. $x^3 + 4x^2 + 3x$ **b.** $x^3 - 6x^2 + 9x$ **c.** $x^3 + 6x^2 + 9x$

2.8 Lesson

Core Vocabulary

factoring by grouping, *p. 108*
factored completely, *p. 108*

Previous
polynomial
binomial

What You Will Learn

- Factor polynomials by grouping.
- Factor polynomials completely.
- Use factoring to solve real-life problems.

Factoring Polynomials by Grouping

You have used the Distributive Property to factor out a greatest common monomial from a polynomial. Sometimes, you can factor out a common binomial. You may be able to use the Distributive Property to factor polynomials with four terms, as described below.

Core Concept

Factoring by Grouping

To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor. This process is called **factoring by grouping**.

EXAMPLE 1 Factoring by Grouping

Factor each polynomial by grouping.

a. $x^3 + 3x^2 + 2x + 6$ **b.** $x^2 + y + x + xy$

SOLUTION

a. $x^3 + 3x^2 + 2x + 6 = (x^3 + 3x^2) + (2x + 6)$ Group terms with common factors.

Common binomial factor is $x + 3$. → $= x^2(x + 3) + 2(x + 3)$ Factor out GCF of each pair of terms.

$= (x + 3)(x^2 + 2)$ Factor out $(x + 3)$.

So, $x^3 + 3x^2 + 2x + 6 = (x + 3)(x^2 + 2)$.

b. $x^2 + y + x + xy = x^2 + x + xy + y$ Rewrite polynomial.

$= (x^2 + x) + (xy + y)$ Group terms with common factors.

Common binomial factor is $x + 1$. → $= x(x + 1) + y(x + 1)$ Factor out GCF of each pair of terms.

$= (x + 1)(x + y)$ Factor out $(x + 1)$.

So, $x^2 + y + x + xy = (x + 1)(x + y)$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial by grouping.

1. $a^3 + 3a^2 + a + 3$ **2.** $y^2 + 2x + yx + 2y$

Factoring Polynomials Completely

You have seen that the polynomial $x^2 - 1$ can be factored as $(x + 1)(x - 1)$. This polynomial is factorable. Notice that the polynomial $x^2 + 1$ cannot be written as the product of polynomials with integer coefficients. This polynomial is unfactorable. A factorable polynomial with integer coefficients is **factored completely** when it is written as a product of unfactorable polynomials with integer coefficients.

Concept Summary

Guidelines for Factoring Polynomials Completely

To factor a polynomial completely, you should try each of these steps.

1.	Factor out the greatest common monomial factor.	$3x^2 + 6x = 3x(x + 2)$
2.	Look for a difference of two squares or a perfect square trinomial.	$x^2 + 4x + 4 = (x + 2)^2$
3.	Factor a trinomial of the form $ax^2 + bx + c$ into a product of binomial factors.	$3x^2 - 5x - 2 = (3x + 1)(x - 2)$
4.	Factor a polynomial with four terms by grouping.	$x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$

EXAMPLE 2 Factoring Completely

Factor (a) $3x^3 + 6x^2 - 18x$ and (b) $7x^4 - 28x^2$.

SOLUTION

a. $3x^3 + 6x^2 - 18x = 3x(x^2 + 2x - 6)$ Factor out $3x$.

$x^2 + 2x - 6$ is unfactorable, so the polynomial is factored completely.

▶ So, $3x^3 + 6x^2 - 18x = 3x(x^2 + 2x - 6)$.

b. $7x^4 - 28x^2 = 7x^2(x^2 - 4)$ Factor out $7x^2$.

$= 7x^2(x^2 - 2^2)$ Write as $a^2 - b^2$.

$= 7x^2(x + 2)(x - 2)$ Difference of two squares pattern

▶ So, $7x^4 - 28x^2 = 7x^2(x + 2)(x - 2)$.

EXAMPLE 3 Solving an Equation by Factoring Completely

Solve $2x^3 + 8x^2 = 10x$.

SOLUTION

$2x^3 + 8x^2 = 10x$ Original equation

$2x^3 + 8x^2 - 10x = 0$ Subtract $10x$ from each side.

$2x(x^2 + 4x - 5) = 0$ Factor out $2x$.

$2x(x + 5)(x - 1) = 0$ Factor $x^2 + 4x - 5$.

$2x = 0$ *or* $x + 5 = 0$ *or* $x - 1 = 0$ Zero-Product Property

$x = 0$ *or* $x = -5$ *or* $x = 1$ Solve for x.

▶ The roots are $x = -5$, $x = 0$, and $x = 1$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Factor the polynomial completely.

3. $3x^3 - 12x$ **4.** $2y^3 - 12y^2 + 18y$ **5.** $m^3 - 2m^2 - 8m$

Solve the equation.

6. $w^3 - 8w^2 + 16w = 0$ **7.** $x^3 - 25x = 0$ **8.** $c^3 - 7c^2 + 12c = 0$

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics

A terrarium in the shape of a rectangular prism has a volume of 4608 cubic inches. Its length is more than 10 inches. The dimensions of the terrarium in terms of its width are shown. Find the length, width, and height of the terrarium.

SOLUTION

1. **Understand the Problem** You are given the volume of a terrarium in the shape of a rectangular prism and a description of the length. The dimensions are written in terms of its width. You are asked to find the length, width, and height of the terrarium.

2. **Make a Plan** Use the formula for the volume of a rectangular prism to write and solve an equation for the width of the terrarium. Then substitute that value in the expressions for the length and height of the terrarium.

3. **Solve the Problem**

$\text{Volume} = \text{length} \cdot \text{width} \cdot \text{height}$	Volume of a rectangular prism
$4608 = (36 - w)(w)(w + 4)$	Write equation.
$4608 = 32w^2 + 144w - w^3$	Multiply.
$0 = 32w^2 + 144w - w^3 - 4608$	Subtract 4608 from each side.
$0 = (-w^3 + 32w^2) + (144w - 4608)$	Group terms with common factors.
$0 = -w^2(w - 32) + 144(w - 32)$	Factor out GCF of each pair of terms.
$0 = (w - 32)(-w^2 + 144)$	Factor out $(w - 32)$.
$0 = -1(w - 32)(w^2 - 144)$	Factor -1 from $-w^2 + 144$.
$0 = -1(w - 32)(w - 12)(w + 12)$	Difference of two squares pattern
$w - 32 = 0$ *or* $w - 12 = 0$ *or* $w + 12 = 0$	Zero-Product Property
$w = 32$ *or* $w = 12$ *or* $w = -12$	Solve for w.

Disregard $w = -12$ because a negative width does not make sense. You know that the length is more than 10 inches. Test the solutions of the equation, 12 and 32, in the expression for the length.

$\text{length} = 36 - w = 36 - 12 = 24$ ✓ *or* $\text{length} = 36 - w = 36 - 32 = 4$ ✗

The solution 12 gives a length of 24 inches, so 12 is the correct value of w.

Use $w = 12$ to find the height, as shown.

$\text{height} = w + 4 = 12 + 4 = 16$

▶ The width is 12 inches, the length is 24 inches, and the height is 16 inches.

Check

$V = \ell wh$

$4608 \stackrel{?}{=} 24(12)(16)$

$4608 = 4608$ ✓

4. **Look Back** Check your solution. Substitute the values for the length, width, and height when the width is 12 inches into the formula for volume. The volume of the terrarium should be 4608 cubic inches.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

9. A box in the shape of a rectangular prism has a volume of 72 cubic feet. The box has a length of x feet, a width of $(x - 1)$ feet, and a height of $(x + 9)$ feet. Find the dimensions of the box.

2.8 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What does it mean for a polynomial to be factored completely?

2. **WRITING** Explain how to choose which terms to group together when factoring by grouping.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, factor the polynomial by grouping. *(See Example 1.)*

3. $x^3 + x^2 + 2x + 2$
4. $y^3 - 9y^2 + y - 9$
5. $3z^3 + 2z - 12z^2 - 8$
6. $2s^3 - 27 - 18s + 3s^2$
7. $x^2 + xy + 8x + 8y$
8. $q^2 + q + 5pq + 5p$
9. $m^2 - 3m + mn - 3n$
10. $2a^2 + 8ab - 3a - 12b$

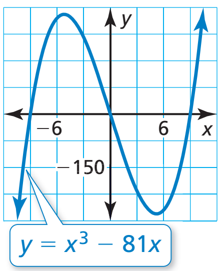
In Exercises 11–22, factor the polynomial completely. *(See Example 2.)*

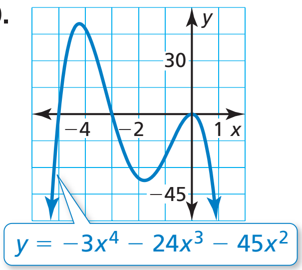
11. $2x^3 - 2x$
12. $36a^4 - 4a^2$
13. $2c^2 - 7c + 19$
14. $m^2 - 5m - 35$
15. $6g^3 - 24g^2 + 24g$
16. $-15d^3 + 21d^2 - 6d$
17. $3r^5 + 3r^4 - 90r^3$
18. $5w^4 - 40w^3 + 80w^2$
19. $-4c^4 + 8c^3 - 28c^2$
20. $8t^2 + 8t - 72$
21. $b^3 - 5b^2 - 4b + 20$
22. $h^3 + 4h^2 - 25h - 100$

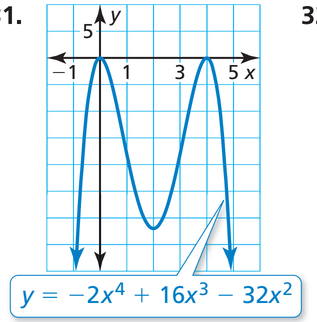
In Exercises 23–28, solve the equation. *(See Example 3.)*

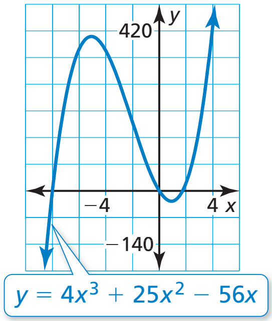
23. $5n^3 - 30n^2 + 40n = 0$
24. $k^4 - 100k^2 = 0$
25. $x^3 + x^2 = 4x + 4$
26. $2t^5 + 2t^4 - 144t^3 = 0$
27. $12s - 3s^3 = 0$
28. $4y^3 - 7y^2 + 28 = 16y$

In Exercises 29–32, find the x-coordinates of the points where the graph crosses the x-axis.

29. 

30. 

31. 

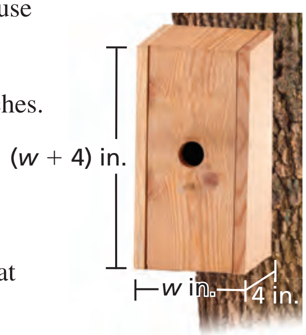
32. 

ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in factoring the polynomial completely.

33. ✗
$$\begin{aligned} a^3 + 8a^2 - 6a - 48 &= a^2(a + 8) + 6(a + 8) \\ &= (a + 8)(a^2 + 6) \end{aligned}$$

34. ✗
$$\begin{aligned} x^3 - 6x^2 - 9x + 54 &= x^2(x - 6) - 9(x - 6) \\ &= (x - 6)(x^2 - 9) \end{aligned}$$

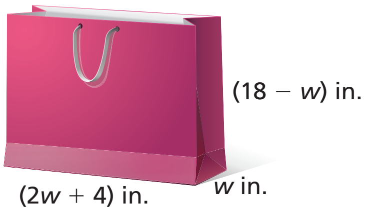
35. **MODELING WITH MATHEMATICS** You are building a birdhouse in the shape of a rectangular prism that has a volume of 128 cubic inches. The dimensions of the birdhouse in terms of its width are shown. *(See Example 4.)*



a. Write a polynomial that represents the volume of the birdhouse.

b. What are the dimensions of the birdhouse?

36. **MODELING WITH MATHEMATICS** A gift bag shaped like a rectangular prism has a volume of 1152 cubic inches. The dimensions of the gift bag in terms of its width are shown. The height is greater than the width. What are the dimensions of the gift bag?



In Exercises 37–40, factor the polynomial completely.

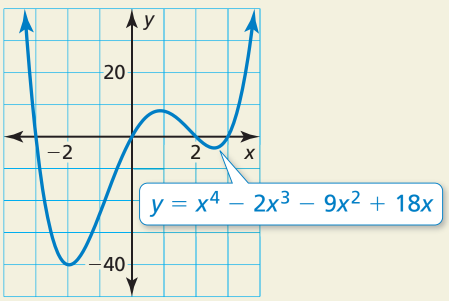
37. $x^3 + 2x^2y - x - 2y$ **38.** $8b^3 - 4b^2a - 18b + 9a$

39. $4s^2 - s + 12st - 3t$

40. $6m^3 - 12mn + m^2n - 2n^2$

41. **WRITING** Is it possible to find three real solutions of the equation $x^3 + 2x^2 + 3x + 6 = 0$? Explain your reasoning.

42. **HOW DO YOU SEE IT?** How can you use the factored form of the polynomial $x^4 - 2x^3 - 9x^2 + 18x = x(x - 3)(x + 3)(x - 2)$ to find the x-intercepts of the graph of the function?



43. **OPEN-ENDED** Write a polynomial of degree 3 that satisfies each of the given conditions.

a. is not factorable **b.** can be factored by grouping

44. **MAKING AN ARGUMENT** Your friend says that if a trinomial cannot be factored as the product of two binomials, then the trinomial is factored completely. Is your friend correct? Explain.

45. **PROBLEM SOLVING** The volume (in cubic feet) of a room in the shape of a rectangular prism is represented by $12z^3 - 27z$. Find expressions that could represent the dimensions of the room.

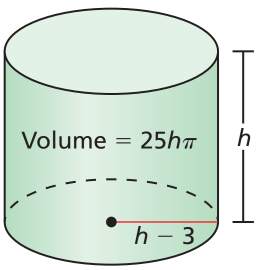
46. **MATHEMATICAL CONNECTIONS** The width of a box in the shape of a rectangular prism is 4 inches more than the height h. The length is the difference of 9 inches and the height.

a. Write a polynomial that represents the volume of the box in terms of its height (in inches).

b. The volume of the box is 180 cubic inches. What are the possible dimensions of the box?

c. Which dimensions result in a box with the least possible surface area? Explain your reasoning.

47. **MATHEMATICAL CONNECTIONS** The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. Find the dimensions of the cylinder.



48. **THOUGHT PROVOKING** Factor the polynomial $x^5 - x^4 - 5x^3 + 5x^2 + 4x - 4$ completely.

49. **REASONING** Find a value for w so that the equation has (a) two solutions and (b) three solutions. Explain your reasoning.

$$5x^3 + wx^2 + 80x = 0$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations by graphing. *(Skills Review Handbook)*

50. $y = x - 4$, $y = -2x + 2$

51. $y = \frac{1}{2}x + 2$, $y = 3x - 3$

52. $5x - y = 12$, $\frac{1}{4}x + y = 9$

53. $x = 3y$, $y - 10 = 2x$

Determine whether the function represents *exponential growth* or *exponential decay*. *(Section 1.6)*

54. $y = \left(\frac{4}{3}\right)^x$

55. $y = 5(0.95)^x$

56. $f(x) = (2.3)^{x-2}$

57. $f(x) = 50(1.1)^{3x}$

2.5–2.8 What Did You Learn?

Core Vocabulary

factoring by grouping, *p. 108*
factored completely, *p. 108*

Core Concepts

Section 2.5

Factoring $x^2 + bx + c$ When c Is Positive, *p. 90*
Factoring $x^2 + bx + c$ When c Is Negative, *p. 91*

Section 2.6

Factoring $ax^2 + bx + c$ When ac Is Positive, *p. 96*
Factoring $ax^2 + bx + c$ When ac Is Negative, *p. 97*

Section 2.7

Difference of Two Squares Pattern, *p. 102*
Perfect Square Trinomial Pattern, *p. 103*

Section 2.8

Factoring by Grouping, *p. 108*
Factoring Polynomials Completely, *p. 108*

Mathematical Practices

1. How are the solutions of Exercise 29 on page 93 related to the graph of $y = m^2 + 3m + 2$?

2. The equation in part (b) of Exercise 47 on page 94 has two solutions. Are both solutions of the equation reasonable in the context of the problem? Explain your reasoning.

Performance Task:

Flight Path of a Bird

Some birds, like parrots, have strong, large beaks to break open nuts and shells. But other birds, like crows, crack open their food by dropping it to the ground. How can a bird change its flight path to protect its falling food from other hungry birds?

To explore the answer to this question and more, check out the Performance Task and Real-Life STEM video at ***BigIdeasMath.com***.

2 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

2.1 Adding and Subtracting Polynomials (pp. 61–68)

Find $(2x^3 + 6x^2 - x) - (-3x^3 - 2x^2 - 9x)$.

$$\begin{aligned}(2x^3 + 6x^2 - x) - (-3x^3 - 2x^2 - 9x) &= (2x^3 + 6x^2 - x) + (3x^3 + 2x^2 + 9x)\\ &= (2x^3 + 3x^3) + (6x^2 + 2x^2) + (-x + 9x)\\ &= 5x^3 + 8x^2 + 8x\end{aligned}$$

Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

1. $6 + 2x^2$ **2.** $-3p^3 + 5p^6 - 4$ **3.** $9x^7 - 6x^2 + 13x^5$ **4.** $-12y + 8y^3$

Find the sum or difference.

5. $(3a + 7) + (a - 1)$

6. $(x^2 + 6x - 5) + (2x^2 + 15)$

7. $(-y^2 + y + 2) - (y^2 - 5y - 2)$

8. $(p + 7) - (6p^2 + 13p)$

2.2 Multiplying Polynomials (pp. 69–74)

Find $(x + 7)(x - 9)$.

$$\begin{aligned}(x + 7)(x - 9) &= x(x - 9) + 7(x - 9) && \text{Distribute } (x - 9) \text{ to each term of } (x + 7).\\ &= x(x) + x(-9) + 7(x) + 7(-9) && \text{Distributive Property}\\ &= x^2 + (-9x) + 7x + (-63) && \text{Multiply.}\\ &= x^2 - 2x - 63 && \text{Combine like terms.}\end{aligned}$$

Find the product.

9. $(x + 6)(x - 4)$ **10.** $(y - 5)(3y + 8)$ **11.** $(x + 4)(x^2 + 7x)$ **12.** $(-3y + 1)(4y^2 - y - 7)$

2.3 Special Products of Polynomials (pp. 75–80)

Find each product.

a. $(6x + 4y)^2$

$$\begin{aligned}(6x + 4y)^2 &= (6x)^2 + 2(6x)(4y) + (4y)^2 && \text{Square of a binomial pattern}\\ &= 36x^2 + 48xy + 16y^2 && \text{Simplify.}\end{aligned}$$

b. $(2x + 3y)(2x - 3y)$

$$\begin{aligned}(2x + 3y)(2x - 3y) &= (2x)^2 - (3y)^2 && \text{Sum and difference pattern}\\ &= 4x^2 - 9y^2 && \text{Simplify.}\end{aligned}$$

Find the product.

13. $(x + 9)(x - 9)$ **14.** $(2y + 4)(2y - 4)$ **15.** $(p + 4)^2$ **16.** $(-1 + 2d)^2$

2.4 Solving Polynomial Equations in Factored Form (pp. 81–86)

Solve $(x + 6)(x - 8) = 0$.

$(x + 6)(x - 8) = 0$	Write equation.
$x + 6 = 0 \quad or \quad x - 8 = 0$	Zero-Product Property
$x = -6 \quad or \quad x = 8$	Solve for x.

Solve the equation.

17. $x^2 + 5x = 0$ **18.** $(z + 3)(z - 7) = 0$ **19.** $(b + 13)^2 = 0$ **20.** $2y(y - 9)(y + 4) = 0$

2.5 Factoring $x^2 + bx + c$ (pp. 89–94)

Factor $x^2 + 6x - 27$.

Notice that $b = 6$ and $c = -27$. Because c is negative, the factors p and q must have different signs so that pq is negative.

Find two integer factors of -27 whose sum is 6.

Factors of -27	$-27, 1$	$-1, 27$	$-9, 3$	$-3, 9$
Sum of factors	-26	26	-6	6

The values of p and q are -3 and 9.

▶ So, $x^2 + 6x - 27 = (x - 3)(x + 9)$.

Factor the polynomial.

21. $p^2 + 2p - 35$ **22.** $b^2 + 18b + 80$ **23.** $z^2 - 4z - 21$ **24.** $x^2 - 11x + 28$

2.6 Factoring $ax^2 + bx + c$ (pp. 95–100)

Factor $5x^2 + 36x + 7$.

There is no GCF, so you need to consider the possible factors of a and c. Because b and c are both positive, the factors of c must be positive. Use a table to organize information about the factors of a and c.

Factors of 5	Factors of 7	Possible factorization	Middle term	
1, 5	1, 7	$(x + 1)(5x + 7)$	$7x + 5x = 12x$	✗
1, 5	7, 1	$(x + 7)(5x + 1)$	$x + 35x = 36x$	✓

▶ So, $5x^2 + 36x + 7 = (x + 7)(5x + 1)$.

Factor the polynomial.

25. $3t^2 + 16t - 12$ **26.** $-5y^2 - 22y - 8$ **27.** $6x^2 + 17x + 7$

28. $-2y^2 + 7y - 6$ **29.** $3z^2 + 26z - 9$ **30.** $10a^2 - 13a - 3$

2.7 Factoring Special Products (pp. 101–106)

Factor each polynomial.

a. $x^2 - 16$

$$x^2 - 16 = x^2 - 4^2 \qquad \text{Write as } a^2 - b^2.$$
$$= (x + 4)(x - 4) \qquad \text{Difference of two squares pattern}$$

b. $25x^2 - 30x + 9$

$$25x^2 - 30x + 9 = (5x)^2 - 2(5x)(3) + 3^2 \qquad \text{Write as } a^2 - 2ab + b^2.$$
$$= (5x - 3)^2 \qquad \text{Perfect square trinomial pattern}$$

Factor the polynomial.

31. $x^2 - 9$ **32.** $y^2 - 100$ **33.** $z^2 - 6z + 9$ **34.** $m^2 + 16m + 64$

2.8 Factoring Polynomials Completely (pp. 107–112)

Factor each polynomial completely.

a. $x^3 + 4x^2 - 3x - 12$

$$x^3 + 4x^2 - 3x - 12 = (x^3 + 4x^2) + (-3x - 12) \qquad \text{Group terms with common factors.}$$
$$= x^2(x + 4) + (-3)(x + 4) \qquad \text{Factor out GCF of each pair of terms.}$$
$$= (x + 4)(x^2 - 3) \qquad \text{Factor out } (x + 4).$$

b. $2x^4 - 8x^2$

$$2x^4 - 8x^2 = 2x^2(x^2 - 4) \qquad \text{Factor out } 2x^2.$$
$$= 2x^2(x^2 - 2^2) \qquad \text{Write as } a^2 - b^2.$$
$$= 2x^2(x + 2)(x - 2) \qquad \text{Difference of two squares pattern}$$

c. $2x^3 + 18x^2 - 72x$

$$2x^3 + 18x^2 - 72x = 2x(x^2 + 9x - 36) \qquad \text{Factor out } 2x.$$
$$= 2x(x + 12)(x - 3) \qquad \text{Factor } x^2 + 9x - 36.$$

Factor the polynomial completely.

35. $n^3 - 9n$ **36.** $x^2 - 3x + 4ax - 12a$ **37.** $2x^4 + 2x^3 - 20x^2$

Solve the equation.

38. $3x^3 - 9x^2 - 54x = 0$ **39.** $16x^2 - 36 = 0$ **40.** $z^3 + 3z^2 - 25z - 75 = 0$

41. A box in the shape of a rectangular prism has a volume of 96 cubic feet. The box has a length of $(x + 8)$ feet, a width of x feet, and a height of $(x - 2)$ feet. Find the dimensions of the box.

2 Chapter Test

Find the sum or difference. Then identify the degree of the sum or difference and classify it by the number of terms.

1. $(-2p + 4) - (p^2 - 6p + 8)$

2. $(9c^6 - 5b^4) - (4c^6 - 5b^4)$

3. $(4s^4 + 2st + t) + (2s^4 - 2st - 4t)$

Find the product.

4. $(h - 5)(h - 8)$

5. $(2w - 3)(3w + 5)$

6. $(z + 11)(z - 11)$

7. Explain how you can determine whether a polynomial is a perfect square trinomial.

8. Is 18 a polynomial? Explain your reasoning.

Factor the polynomial completely.

9. $s^2 - 15s + 50$

10. $h^3 + 2h^2 - 9h - 18$

11. $-5k^2 - 22k + 15$

Solve the equation.

12. $(n - 1)(n + 6)(n + 5) = 0$

13. $d^2 + 14d + 49 = 0$

14. $6x^4 + 8x^2 = 26x^3$

15. The expression $\pi(r - 3)^2$ represents the area covered by the hour hand on a clock in one rotation, where r is the radius of the entire clock. Write a polynomial in standard form that represents the area covered by the hour hand in one rotation.

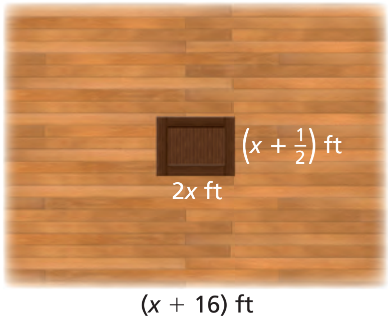
16. A magician's stage has a trapdoor.

a. The total area (in square feet) of the stage can be represented by $x^2 + 27x + 176$. Write an expression for the width of the stage.

b. Write an expression for the perimeter of the stage.

c. The area of the trapdoor is 10 square feet. Find the value of x.

d. The magician wishes to have the area of the stage be at least 20 times the area of the trapdoor. Does this stage satisfy his requirement? Explain.



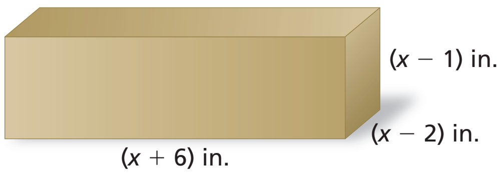
17. Write a polynomial equation in factored form that has three positive roots.

18. You are jumping on a trampoline. For one jump, your height y (in feet) above the trampoline after t seconds can be represented by $y = -16t^2 + 24t$. How many seconds are you in the air?

19. A cardboard box in the shape of a rectangular prism has the dimensions shown.

a. Write a polynomial that represents the volume of the box.

b. The volume of the box is 60 cubic inches. What are the length, width, and height of the box?



2 Cumulative Assessment

1. Classify each polynomial by the number of terms. Then order the polynomials by degree from least to greatest.

a. $-4x^3$

b. $6y - 3y^5$

c. $c^2 + 2 + c$

d. $-10d^4 + 7d^2$

e. $-5z^{11} + 8z^{12}$

f. $3b^6 - 12b^8 + 4b^4$

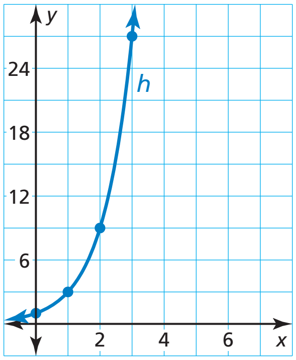
2. Which exponential function has the greatest percent rate of change?

Ⓐ $f(x) = 4(2.5)^{x/3}$

Ⓑ

x	0	1	2	3	4
$g(x)$	8	12	18	27	40.5

Ⓒ



Ⓓ An exponential function j models a relationship in which the dependent variable is multiplied by 6 for every 1 unit the independent variable increases. The value of the function at 0 is 2.

3. Find the x-coordinates of the points where the graph of $f(x) = x^2 - 2x - 24$ crosses the x-axis.

-24	-6	-4	-2	-1	0	1	2	4	6	24

4. The table shows your distances from a national park over time.

a. Write and graph an absolute value function that represents the distance as a function of the number of hours.

b. When do you reach the national park? Explain.

c. Write the function in part (a) as a piecewise function.

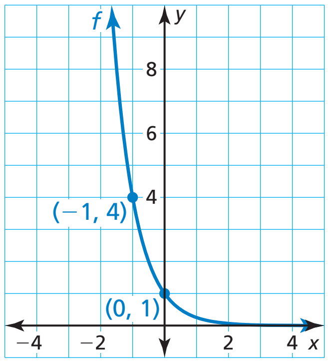
Hours, x	Distance (miles), y
1	195
2	130
3	65
4	0
5	65
6	130
7	195

5. Which function has a steeper graph, $f(x) = 2x - 3$ or the inverse of f? Explain your reasoning.

6. Which expressions are equivalent to $-2x + 15x^2 - 8$?

$15x^2 - 2x - 8$	$(5x + 4)(3x + 2)$
$(5x - 4)(3x + 2)$	$15x^2 + 2x - 8$
$(3x - 2)(5x - 4)$	$(3x + 2)(5x - 4)$

7. The graph represents an exponential function.



a. Write the function.

b. Find $f\left(\frac{1}{2}\right)$ and $f\left(-\frac{5}{2}\right)$.

8. Which polynomial represents the product of $3x + 1$ and $4x - 1$?

Ⓐ $7x - 1$

Ⓑ $12x^2 - 1$

Ⓒ $12x^2 - x - 1$

Ⓓ $12x^2 + x - 1$

9. You are playing miniature golf on the hole shown.

a. Write a polynomial that represents the area of the golf hole.

b. Write a polynomial that represents the perimeter of the golf hole.

c. Find the perimeter of the golf hole when the area is 216 square feet.